Quantum Nyquist Temperature Fluctuations

A. V. Balatsky [a,1], Jian-Xin Zhu [a]

[a] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

Abstract

Using the Nyquist approach, temperature fluctuations of an object, in thermal contact with a reservoir, are studied. We argue that, upon decreasing the size of the object, one necessarily reaches the quantum regime. The crossover temperature between the classical and quantum regimes is given by $T^* \sim \hbar/k_B\tau$, where τ is the thermal relaxation time of the system. For a nano-scale metallic particle in a good thermal contact with a reservoir, T^* can be on a scale of a few Kelvin.

Key words: Quantum fluctuations; temperature; theromodynamics; nano-scale systems.

In recent years, there has been increasing interest in the nano-scale problems [1,2]. For these nano-scale systems, the temperature fluctuation can be large. The existing study has been limited to the classical regime. Any classical variable has its corresponding standard quantum limit where quantum fluctuations dominate. Similarly, we expect temperature T will have its quantum limit. Here we argue that when the temperature is below $T^* \sim \hbar/\tau$, where τ is the thermal relaxation time of the nano-scale particle, a quantum temperature fluctuation regime emerges.

Consider a set of quantum dots, as shown in Fig. 1. Assume these dots to be similar in the number of contained particles, size, etc. In addition, each dot has discrete levels, which are filled by a sufficient number ($N \gg 1$) fermions (e.g., electrons) or bosons (e.g., ^{4}He atoms). All these dots are in contact with a substrate (large plate) which plays the role of a thermal reservoir. The reservoir is kept at a certain temperature T. The thermal contact between the dots and the reservoir will cause the thermal fluctuations in the dots. As a result, the heat flows to/from the reservoir. The relaxation time for the thermal process between the dots and the reservoir is τ.

[1] Corresponding author. Fax: (505) 6654063; E-mail: avb@viking.lanl.gov

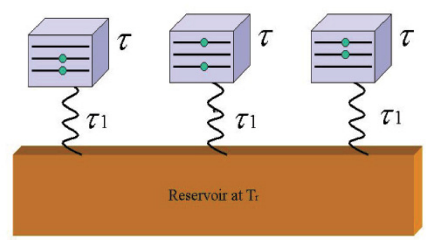

Fig. 1. The ensemble of the quantum dots.

We use the Nyquist approach [3] to treat the temperature fluctuation. Assume now temperature T to play the role of a generalized coordinate and the entropy S the role of the generalized fluctuating force. The relaxation process of the temperature can be described by a linearized macroscopic "equation of motion":

$$\frac{d\Delta T}{dt} = -\lambda(\Delta T - \frac{\partial T}{\partial S}|_{\bar{T}_0}\Delta S) \;, \tag{1}$$

where $\lambda = 1/\tau$, and $\Delta T = T - \bar{T}_0$, and is the deviation of the equilibrium temperature T as a result of the fluctuating force ΔS. Performing the Fourier transform for ΔT and ΔS, we arrive at $\Delta T_\omega = \alpha(\omega)\Delta S_\omega$, with $\alpha(\omega) = \frac{\lambda T}{C_v(-i\omega+\lambda)}$. Using the Callen-Welton fluctuation-dissipation theorem [4], it immediately follows:

$$\langle \Delta T^2 \rangle_\omega = \hbar \coth(\hbar\omega/2k_B T)\alpha''(\omega) \ , \quad (2)$$

where the imaginary part of $\alpha(\omega)$: $\alpha''(\omega) = \frac{\lambda T}{C_v}\frac{\omega}{\omega^2+\lambda^2}$. For the average quadratic fluctuation of T, it can be found:

$$\langle \Delta T^2 \rangle = \frac{\hbar\lambda T}{2\pi C_v}\int_{-\infty}^{\infty} d\omega \frac{\omega}{\omega^2+\lambda^2}\coth(\hbar\omega/2k_B T) \ . \quad (3)$$

When $k_B T/\hbar\lambda \gg 1$, we have

$$\langle \Delta T^2 \rangle = \frac{k_B T^2}{C_v}\left[1 + \frac{\hbar\lambda}{\pi k_B T}\ln\frac{\hbar\omega_c}{k_B T}\right] \ , \quad (4)$$

where we have introduced an upper band cutoff $\omega_c \sim 1/\tau_1$ on the order of the relevant bandwidth. One can recognize immediately that Eq. (4) is the classical limit of the temperature fluctuations. In the opposite limit, $\hbar\lambda \gg k_B T$, one finds:

$$\langle \Delta T^2 \rangle = \frac{\hbar\lambda T}{\pi C_v}\ln\frac{\omega_c}{\lambda} \ , \quad (5)$$

which means that at low temperatures, the fluctuations would acquire a distinctly quantum character with $\hbar/\tau$ entering into the magnitude of $\langle \Delta T^2 \rangle$. Any fluctuation, described by Eq.(5), happen on a characteristic time scale τ. The high temperature expansion in Eq. (4) has already indicated the crossover temperature

$$T^* = \frac{\hbar\lambda}{k_B\pi}\ln\frac{\omega_c}{\lambda} \ , \quad (6)$$

at which there is a change of the regime from the classical to quantum fluctuations. Figure 2 displays schematically the temperature dependence of the fluctuations δT. Physically, $T^* \approx \hbar/k_B\tau$ corresponds to the uncertainty in energy associated with the relaxation process in the subsystem. The reservoir is attached to a subsystem via a thermal contact that has its own bandwidth $\hbar/\tau$ and any temperature fluctuation will relax on the scale of τ. Once $T \ll T^*$, the intrinsic bandwidth of the contact rather than the temperature will dominate the Gaussian fluctuations.

The low temperature limit $T \ll \hbar\lambda/k_B$ implies that the relaxation time of the thermal object has to be short enough. Since the thermal relaxation time $\tau = C_v R_T$, where R_T is the thermal resistance of the contact between the object and the thermal reservoir. For the metallic system— a nanometer mechanical resonator, which is a cylindrical gold (Au) rod of 1 μm in length L and 15 nm in radius r. One then finds $\tau \sim$ 6.7psec, and $T^* \sim 1$ K, respectively, which is now experimentally accessible. For the case of a small bosonic system— a droplet of ^{4}He of size 0.1 μm, which is enclosed in a metallic container such as lead. Using the values for C_v and R_T from Fig. 8.6 in Ref. [5], One then obtains $\tau \sim 6.9 \times 10^{-8}$ second, and $T^* \sim 10^{-3}$K, which is small for the given size of the droplet.

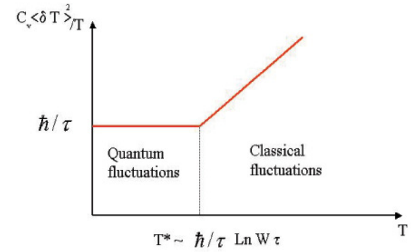

Fig. 2. Temperature dependence of the fluctuations δT.

Experimentally the proposed crossover to quantum regime can be seen as a change in temperature dependence of noise of some observable. The choice depends on a specifics of the experiment obviously, e.g for an oscillating clamped beam [2] it can be a noise of the mechanical oscillatior. In the case of magnetization noise [6], one would desire to measure noise in the SQUID at relevant frequencies $1/\tau$.

In summary, we have shown for the first time that when at temperatures below a characteristic value $T^* \sim \hbar/k_B\tau$, the temperature fluctuation would acquire a distinctly quantum character. In light of recent advances in nano-technology, the quantum fluctuation regime should be experimentally accessible and might be relevant for the experiments on nanoscale systems.

Acknowledgements

This work was supported by the Department of Energy.

References

[1] A. N. Cleland and M. L. Roukes, Nature **392**, 160 (1998).

[2] A. N. Cleland and M. L. Roukes, unpublished.

[3] H. Nyquist, Phys. Rev. **32**, 110 (1928).

[4] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).

[5] J. Wilks, *An Introduction to Liquid Helium*, 2nd edition (Clarendon Press, Oxford, 1987).

[6] T. C. P. Chui, D. R. Swanson, M. J. Adriaans, J. A. Nissen, and J. A. Lipa, Phys. Rev. Lett. **69**, 3005 (1992).