Resistance Induced by Quantum Phase-Slips in Superconducting Nanowires

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Abstract

We have measured the resistive transitions of ~ 20 nanowires of superconducting amorphous MoGe with diameters ~ 10 nm and lengths from 100 nm to 1000 nm. The transition width increases with decreasing cross sectional area (i.e. increasing normal resistance per unit length) as described by the phenomenology including quantum phase-slips used by Giordano to explain his earlier data and also by the rather similar results of a microscopic theory of Golubev and Zaikin. The resistance well below T_c is much greater than can be explained by thermally activated phase-slips alone. We consider this to strongly support the reality of quantum phase-slips, and the basic correctness of these theories. The exact role of dissipation, whether from metal in the wire, from the carbon nanotube substrate, or from the electromagnetic environment, in reducing quantum phase-slips needs further clarification.

1. Introduction

Superconducting long-range order and zero resistance are forbidden by the Mermin-Wagner theorem [1] in the limit of a strictly one-dimensional system. In this paper, we review the progress that has been made in unraveling how this limit is approached in physical wires of superconducting material as they are made smaller and smaller in diameter. The first big step was the theoretical work of Langer and Ambegaokar and of McCumber and Halperin (LAMH) [2]. As proposed earlier by Little, they showed that the elementary process giving rise to resistance in a thin superconducting wire was a "phase-slip", in which the magnitude of the superconducting order parameter locally and momentarily fluctuates to zero, allowing the phase difference of the order parameter to "rotate" by 2π before the magnitude relaxes back to its full value. Such a 2π phase slip is associated by the Josephson relation with a voltage pulse whose time integral is $h/2e = 2\times 10^{-15}$ volt-sec. In the presence of a current, more of the slips are of one sign than of the other, and a net dc resistive voltage results, proportional to the current. The LAMH analysis yielded an explicit formula for the thermal activation energy for such a process

$$\Delta F = (8\sqrt{2}/3)(H_c^2/8\pi)A\xi$$

and showed that the attempt frequency was approximately given by

$$\Omega \approx (1/\tau_{GL})L/\xi,$$

where $1/\tau_{GL}$ is the Ginzburg-Landau relaxation rate (which is of the order of the energy gap frequency), ξ is the coherence length, and L is the length and A the cross-sectional area of the wire. This value for ΔF represents the loss of superconducting condensation energy over a wire length $\sim \xi$, and the value for Ω reflects the existence of independent fluctuations in segments of length ξ along the wire. This very plausible result quickly was confirmed by experiments at Cornell and Harvard [3] on tin whiskers of diameter $\sim 0.5\mu m$. These showed that the resistive transition followed an exponential temperature dependence with a width of order 1 mK, as predicted by the LAMH calculation for a wire of that cross-sectional area.

Two decades later, Giordano [4] was able to fabricate wires of considerably narrower cross-section, and he repeated these measurements on wires of several materials. Given their narrower cross-section than the tin whiskers, the transition widths were considerably greater than in the earlier experiments, as expected, but, surprisingly, they also showed a slower drop in resistance at lower temperatures, whereas any thermal activation model must give an ever *steeper* drop in resistance because of the $1/T$ factor in the exponent. To account for this, Giordano proposed a phenomenological model which included quantum phase-slips which do not "freeze out" at low temperatures in addition to the thermal ones treated by LAMH. This proposal remained controversial, however, because sample granularity, leading to weak links between grains, could also account for a somewhat similar T-dependence of resistance.

2. Recent Work

Recently, our group has revisited this problem, using a novel technique due to Bezryadin [5] to fabricate wires of the superconductor $MoGe$ which are considerably narrower and more uniform than those studied earlier. The essential new technique is the use of a freely suspended carbon nanotube (or rope of them) as a template to determine the width of the metal layer deposited on top of it. In this way wires as narrow as $\sim 5nm$ in width and of similar thickness could be formed. Taking advantage of the homogeneity of this well-studied amorphous material, the cross sectional area could be determined from the measured length and resistance, instead of the cross sectional dimensions, which are not as accurately known.

Because of the exponential variation of resistance with the activation energy ΔF, which is, in turn, proportional to the cross-sectional area A, our data on some 20 samples with a $\sim 4:1$ range in A shows a wide range of transition widths and shapes, as can be seen in Fig. 1(a). This simple plot of R vs. T for a number of samples shows a confusion of crossing curves, because these samples range in length by a factor of 10 as well as in cross-section by a factor of 4, both of which enter into determining the measured resistance. To sort this out, in Fig. 1(b) we plot the same data, now as resistance *per unit length*, presuming that any intrinsic length dependence is less important than the expected exponential dependence on area. We also normalize the temperature scale to the critical temperature of the bulk film to eliminate any dependences not related to the narrow wire. When plotted in this way, the $R(T)$ curves show a relatively orderly progression from essentially temperature-independent for the thinnest wires (those with $R_n/L \approx 100 ohms/nm$) to those showing a rapid fall in resistance with decreasing temperature for wires with ~ 3 times greater cross-section, such as ones with $R \sim 30 ohms/nm$ at T_c. The whole range of these data were well accounted for, using the theoretically predicted T-dependence based on the Giordano phenomenology, as reported in [6].

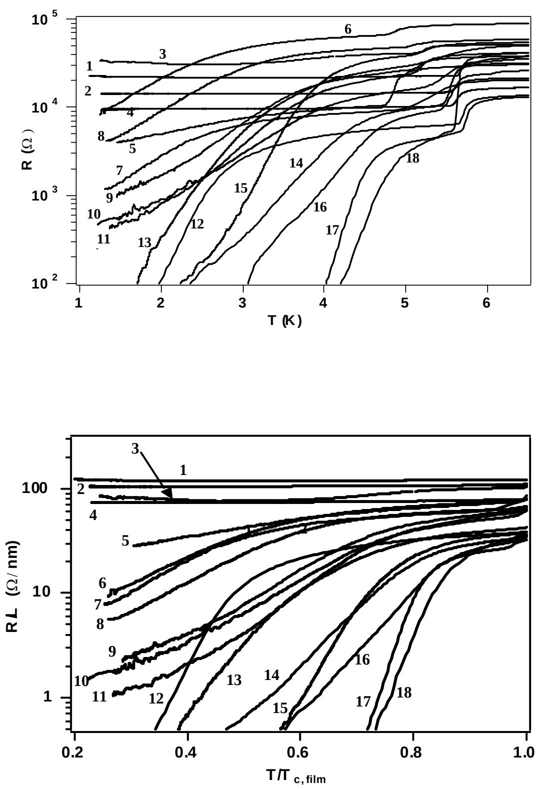

Fig. 1. (a). R(T) for 18 different samples. (b) Resistance per unit length as a function of normalized temperature for the same set of samples.

At about the same time, Golubev and Zaikin [7] published a detailed microscopic analysis of the probability of quantum phase-slips at $T = 0$, obtaining results very similar to those from the Giordano phenomenological formulation. In Fig. 2, we show a comparison of their results with the experimental data. Since their analysis does not give an explicit temperature dependence, we interpolate between $T = 0$ and T_c using the same $(1-t)^{1/2}$ dependence for the exponent describing quantum phase-slips as was used in the Giordano phenomenology.

In making this comparison, the only sample-dependent parameter is the measured R_n/L. There are also two parameters of order unity which are the same for all samples. Although the detailed agreement is far from perfect, it is impressive how well the sample dependence is captured with only a single, measured parameter. This gives us considerable confidence that the quantum phase-slip term, which dominates below roughly $1/2\ T_c$, is, in fact, being observed. *Any* model involving *only* thermally activated processes would inevitably give an $R(T)$ dependence that drops faster

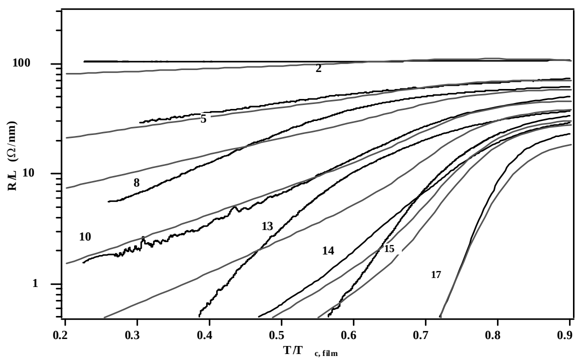

Fig. 2. Comparison of data (solid curves) on 8 representative samples with the predictions of Golubev-Zaikin theory (dotted curves). The only free parameters are of order unity: $a_{GZ} = 0.43$ and $B_{GZ} = 2.5$ for the whole family of curves.

and faster as T is lowered, and could not fit the data on the thinner wires.

Although this general agreement of the data with these simple models is heartening, two disagreements deserve comment. Firstly, in a few cases two of the experimental curves for samples of similar cross-section cross when plotted vs. T. This could not happen if there were truly a single parameter characterizing each sample. However, we have shown by simulations that if the diameter of the wire fluctuates randomly with a Gaussian width of $\pm 1.5nm$ along the length of the wire, this is sufficient to account for such a crossing. Such a fluctuation is consistent with TEM photos of similar wires. The second, and related, disagreement is that not all the experimental curves have exactly the same "shape". There appear to be two "classes": those showing a transition from negative to positive curvature with decreasing temperature, as predicted by the theory, and those that do not. This difference could readily account for a crossing between two curves from different classes. As yet, we have not established a convincing explanation for the two classes, but we are exploring the possibility that the two classes may correspond to samples deposited on metallic vs. semiconducting carbon nanotubes, which would be expected to give different amounts of damping for the phase-slip process. This would have little effect on the thermal phase slips nearer to T_c, but extra damping would be expected to reduce the rate of the macroscopic quantum tunneling process responsible for the quantum phase-slips that dominate at low temperatures.

This possibility raises the whole question of the role of damping in accounting for the measurements. In general, damping strongly reduces the probability of quantum phase slips because it couples the microscopic system to the larger classical environment. In the simpler zero-dimensional system of a Josephson junction (JJ) shunted by a damping resistance, Schmid [8] showed theoretically that if the damping resistor R was smaller than the quantum resistance $R_q = h/4e^2 \approx 6.5kohm$, the phase should become localized at $T = 0$ and the system would show no voltage or resistance, regardless of the magnitude of the Josephson coupling energy E_J. The Helsinki group [9] has devoted considerable effort to testing this result for a single JJ, and has found evidence for its correctness in junctions with small E_J, but for larger values of E_J the resistance predicted by ordinary fluctuation theory without consideration of localization is so small that experimentally it is hard to detect the difference from zero.

An outstanding open question at this time is how this localization argument might carry over to the case of a wire. It seems likely that the phase-slip energy ΔF would be the analog of E_J, but by analogy with the JJ case, the damping resistance, not E_J, should be the critical parameter. What is the appropriate damping resistance for a long wire? The entire resistance of the wire, as suggested in [5], now seems less likely; another possibility is the normal resistance of some length comparable to ξ , which is the length scale of the phase-slip process that is the analog of the JJ. In that case, the damping conductance would have the same dependence on sample parameters as the energy barrier ΔF, and it would be hard to distinguish an exponential dependence on ΔF from one on a damping conductance. In addition to dissipation in the metallic wire itself, there can be external dissipation coupled electromagnetically. This issue could be clarified experimentally by adding a controllable damping conductance in parallel with the wire, which would allow the damping to be varied independently of the energy barrier. Such experiments are planned, but we have no results to report at this time.

A semantic issue is the definition of "superconductivity" in a thin wire. In the previous paragraph, we discussed the issue of whether the resistance would be zero at the unattainable $T = 0$ because of damping, even if the energy barrier to phase slip by itself would only give an exponentially small, but finite, resistance. A much more operational, but different, criterion for superconductivity is simply whether the resistance decreases significantly when the temperature is reduced below T_c. This question can be posed theoretically by comparing the superconducting resistance due to quantum phase slips at $T = 0$ with the normal resistance at T_c. Using our simple phenomenological expression for quantum phase slip resistance at $T = 0$, we find [10] that $R(0)$ should be less than $R(T_c)$ if the normal resistance of a length of wire approximately 13 ξ long is less than the quantum resistance. For the coherence length of 5 nm appropriate to $MoGe$, this implies a resistance/length of about 100 ohms/nm, which is in good agreement with the data in Fig. 1(b). This suggests that the energy barrier to phase slips controls $R(T)$ over most of the range, whereas dissipation may control the limiting behavior at $T \approx 0$.

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