

# Noise Effect on Resonant Tunneling in the Nanoscale Molecular Magnet (Fe<sub>8</sub>)

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## Abstract

We study the effect of noise on the resonant tunneling for a realistic model of Fe<sub>8</sub>. We discuss both the static and dynamical effect of noise on the estimation of tunnel splitting by the use of the Landau-Zener-Stückelberg (LZS) formula.

*Key words:* Fe<sub>8</sub>; resonant tunneling; tunnel splitting; noise

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## 1. Introduction

Since nanoscale molecular magnets (Mn<sub>12</sub>, Fe<sub>8</sub>, etc.) were reported to exhibit unprecedented features, much effort has been exerted to the quantum tunneling of magnetization. The tunneling dynamics has been studied from the viewpoint of the nonadiabatic transition [1–3]. The transition probability in the two level system is given by the Landau-Zener-Stückelberg (LZS) formula [4]. This formula is useful to pure quantum dynamics. However, in real systems fluctuation and dissipation affect the process of quantum dynamics. At low enough temperatures, effects of the ubiquitous hyperfine field within the molecule and the dipole field of other molecules become significant for the relaxation process. Therefore, we have to take notice of estimating the tunnel splitting by the use of LZS formula. We have already studied the effect of noise in the case  $S = 1/2$  and clarified that the effect of noise cannot be negligible and the estimated gap deviates from the true tunnel splitting [5]. Here we study the noise effect of a realistic model with  $S = 10$  for Fe<sub>8</sub> [6–8]. It is found that the estimated gap coincides with the true gap in the fast sweeping regime, while the effect of noise cannot be negligible in the

slow sweeping regime.

## 2. Estimation of the tunnel splitting

We consider the following Hamiltonian which is adopted for the molecular magnet of Fe<sub>8</sub>.

$$\begin{aligned}\hat{H}(t) = & -D(S^z)^2 + E((S^x)^2 - (S^y)^2) \\ & + C((S^+)^4 + (S^-)^4) - \tilde{h}(t)S^z \\ & + \hat{H}^{\text{noise}}(t),\end{aligned}\tag{1}$$

where we set  $D = 0.292(\text{K})$ ,  $E = 0.046(\text{K})$ , and  $C = -2.9 \times 10^{-5}(\text{K})$  [6]. The field  $(\tilde{h}(t))$  in the z-direction is swept linearly with time. We calculate the tunneling probability under the influence of the noise. We adopt  $\hat{H}^{\text{noise}}(t) = h_\alpha(t)S^\alpha$ , where  $\alpha$  denotes a direction of  $x$ ,  $y$ , or  $z$ . Here  $h(t)$  is a random gaussian noise and has an exponential-decaying autocorrelation. We define  $B$  as the amplitude of the noise [5]. Using the LZS formula, the tunneling probability  $p$  between the magnetizations  $m$  and  $m'$  in the sweeping field is given as a function of the energy gap  $\Delta E_{m,m'}$  and the sweeping velocity  $c(= \frac{dH_z}{dt})$ :  $p = 1 - \exp(-\frac{\pi(\Delta E_{m,m'})^2}{2\hbar|m-m'|c})$ . In the experiments,  $p$  was estimated for given  $c$  and the tunnel splitting was estimated as a function of the velocity making use this formula. Here we estimate the effect of noise on estimation of the tunnel splitting by evaluating the following quantity.

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$$\Delta \equiv \frac{\Delta E_{\text{eff}}}{\Delta E_{m,m'}}, \quad \Delta E_{\text{eff}} \equiv \sqrt{-\frac{2\hbar|m-m'|c\ln(1-p)}{\pi}}. \quad (2)$$

$\Delta E_{m,m'}$  is the true energy gap and  $\Delta E_{\text{eff}}$  is the estimated energy gap from the obtained values of  $c$  and  $p$ . We can evaluate the effect of noise by estimating how much  $\Delta$  deviates from 1.

### 3. Results and Discussion

In the beginning, we focus on the regime of the fast sweeping because the estimated gap shows no velocity dependence in the experiments [7,8]. In this regime, the noise could be considered to be static [5]. The tunnel splitting  $\Delta E_{-10,10}$  is estimated numerically as  $0.452 \times 10^{-7}$  (K) for the system (1).

We estimate numerically the dependence of the gap on static noise  $h_x$  and  $h_y$ , and approximate them by an algebraic function  $f(h) = \Delta E_{-10,10} + \alpha h^2$ , where  $\alpha = -1.356 \times 10^{-6}$  (K/T<sup>2</sup>) for  $h_x$  and  $\alpha = 2.056 \times 10^{-6}$  (K/T<sup>2</sup>) for  $h_y$ . The error bar of  $\alpha$  is within  $10^{-11}$ . The values of the gap  $\Delta E$  in the presence of  $h_y$  and the fitting function  $f(h_y)$  are shown in Fig. 1. We find that  $0.997 \leq \Delta_x \leq 1$  and  $1 \leq \Delta_y \leq 1.005$  [9] when  $B = \sqrt{\langle h_y \rangle^2} \leq 0.1 \times 10^{-1}$  (T), whose energy scale is  $10^6$  times as large as the tunnel splitting  $\Delta E_{-10,10}$ . This holds true from  $c = 10^{-5}$  to  $c = 10^0$  (T/s), where the experiments were performed. This indicates that the estimated gap is equal to the true gap. This is because the transition from  $m = -10$  to  $m' = 10$  is the 20th perturbation process and the term of  $h_x S^x$  or  $h_y S^y$  can give only little effect on the transition process.

Next we investigate the dynamical effect of the noise. It is difficult to simulate time-dependent Schrödinger equation for the system (1) with realistic parameters. However the flowing parameter set is enough to investigate the dynamical effect of 20th perturbation:  $D = 0.1$ ,  $E = 0.07$ , and  $C = 0$ . The tunnel splitting ( $\Delta E_{-10,10}$ ) is 0.00113 and we set  $B = \Delta E_{-10,10}/40$ . We show the sweeping rate dependence of  $\Delta_x$  in Fig. 2, where  $\hbar = 1$ .  $\gamma$  is the damping factor of the noise. Here the arrow of Fig. 2 shows the regime of sweeping rate corresponding to that of the experiment. It is found that the fluctuation of noise has an influence on the tunneling process and the estimated gap decreases in the slow sweeping region even if  $B$  is not so large. This tendency reproduces the experimental results qualitatively.

The system  $S = 1/2$  is sensitive to the effect of noise and the estimated gap should be corrected by  $O(\frac{B^2}{\Delta E^2})$  for the fast sweeping regime. On the other hand, the effect of noise can be ignored in the estimation of the tunnel splitting in the case  $S = 10$ . It is mainly due to the 20-th perturbation. However the fluctuation of noise gives an influence to the tunneling process in the

slow sweeping regime regardless of the amplitude of noise. As a result, reduction of the estimated gap is observed in that regime, which reproduces the corresponding experimental data qualitatively.

In the experimental results, isotopically substituted Fe<sub>8</sub>s show the different estimated values [7,8]. This difference cannot be explained within our modeling. A treatment from a more microscopic viewpoint may be necessary.

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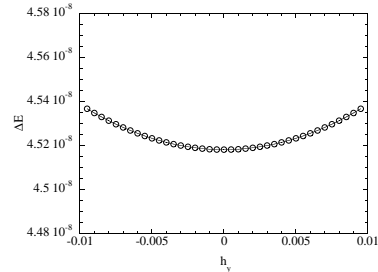


Fig. 1.  $\Delta E(h_y)$  (○) and  $f(h_y)$  (Solid line)

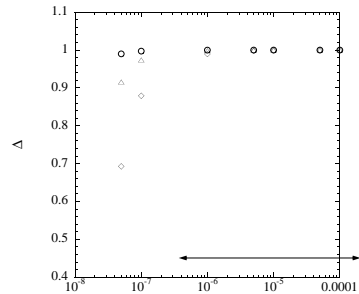


Fig. 2. Sweeping rate dependence of  $\Delta_x$ . ○( $\gamma=0.01$ ), △(0.1). 1.0(◇).