

# New Evaluation Method for Optical Conductivity of Superconducting Thin Films

Etsuo Kawate,<sup>a,1</sup> Yoshinori Uzawa<sup>b</sup>, Zhen Wang<sup>b</sup>

<sup>a</sup>*National Institute of Advanced Industrial Science and Technology, Tsukuba Central 2, Tsukuba, Ibaraki 305-8588, JAPAN*

<sup>b</sup>*Kansai Advanced Research Center, Communication Research Laboratory Iwaoka-cho, Nishi-ku, Kobe 651-2401, JAPAN*

---

## Abstract

A symmetry X optical setup has been developed for the absolute measurements of reflectance and transmittance of a specular sample. Both the transmittance spectra and the geometric mean of the reflectance spectra were measured within the uncertainty of  $\pm 0.4\%$ , respectively. The solution of two simultaneous equations with the measured reflectance and transmittance determines the optical conductivity of the sample. The conductivity by the new method is compared with that by the ordinary method using Kramers-Kronig relations.

**Key words:** absolute reflectance; absolute transmittance; optical conductivity; Kramers-Kronig relations

---

## 1. Introduction

Reflectance ( $R$ ) of an ideal BCS-type superconductor is approaching to unity and transmittance ( $T$ ) is also approaching to zero in the small frequency region at sufficiently low temperature. However, the sum of  $R$  and  $T$  of high  $T_c$  superconductors is generally less than unity ( $R + T < 1$ ) and absorptance ( $A$ ) always remains. The absorptance is due to  $d$ -wave superconductors. The technique to measure the absolute absorptance with high accuracy is important to study the high  $T_c$  superconductors.

In the optical measurement the law of conservation of energy of a sample with a specular surface is  $R + T + A = 1$ . These three parameters are considered to be a function of complex optical conductivity ( $\sigma = \sigma_1 + i\sigma_2$ ). When two parameters are measured at any frequency and these two simultaneous equations are solved, the conductivity can be uniquely determined at the frequency (R-T method) [1].

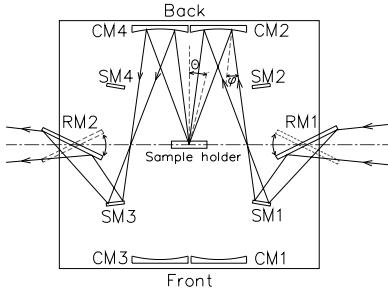


Fig. 1. A symmetry X optical setup. Four CM's are concave mirrors and the others are flat mirrors.

## 2. Experimental

A schematic layout of the symmetry X optical setup is shown in Fig. 1 [2]. It included ten mirrors, two automatic rotation stages (RM1 and RM2), an automatic z-axis stage under a sample holder and a space for a cryostat. It was installed in a sample compartment of a usual FT-IR spectrometer without any changes of the spectrometer itself. In order to measure absolute  $R$  and  $T$ , the sample holder in Fig. 1 had two holes of the same size. One was an empty hole and the other

---

<sup>1</sup> E-mail: e.kawate@aist.go.jp

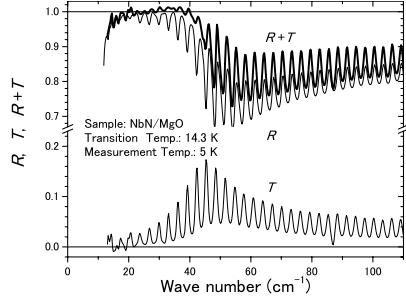


Fig. 2. Reflectance, transmittance and sum to two spectra

was covered with the sample. Two transmittances and two reflectances were measured by front and back incidences to the sample using RM1, RM2 and z-axis stages. A layout of a reflection signal by the back incidence is shown in Fig. 1.

Samples were an undoped silicon (Si) and a NbN thin film on MgO substrate (NbN/MgO). The thickness of Si, MgO and the thin film was 0.38 mm, 0.5 mm and 41 nm, respectively.

### 3. Results and discussion

In order to evaluate the symmetry X optical setup, absolute  $R$  and  $T$  of the undoped Si were measured in the near infrared region ( $2,000 \sim 5,000 \text{ cm}^{-1}$ ), because Si is well known to be transparent in this region. Two measured transmittances and calculated transmittance from the complex refractive index ( $n$  and  $k$ ) in handbooks agreed within  $\pm 0.4\%$  and the geometric mean of two measured reflectances and calculated reflectance also agreed within  $\pm 0.4\%$  [2]. Although the ten mirrors were used in this setup, it was disclosed that the geometric reflectance was independent of the reflectance difference of the individual mirrors [2].

NbN/MgO is well known as a BCS-type superconductor. Absolute  $R$  and  $T$  of NbN/MgO were measured in the far infrared region ( $10 \sim 200 \text{ cm}^{-1}$ ), because MgO is nearly transparent at low temperature.  $R$ ,  $T$  and  $R + T$  spectra with high resolution are shown in Fig. 2. The fringe is due to multiple reflections inside the MgO substrate.  $R$  is approaching to unity and  $T$  is also approaching to zero in the small frequency region. But  $R + T$  is larger than unity in the region. It is thought that the measured  $T$  is larger than real  $T$  due to the non-linear sensitivity of Si-bolometer. The strict expressions of  $R$  and  $T$  as a function of  $n$  and  $k$ , the thickness and the frequency are complicated, so we approximated these expressions in the case of metal ( $n, k \gg 1$ ). These two simultaneous equations with the measured reflectance and transmittance can be easily solved. The solutions uniquely determined the optical

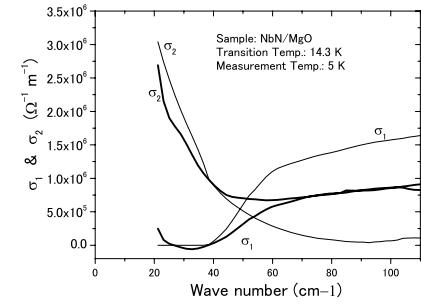


Fig. 3. Optical conductivity of NbN thin film. Thick curves obtained by R-T method and thin curves by K-K method.

conductivity as shown in Fig. 3 by thick solid curves.

On the other hand, the optical conductivity can also be extracted from the reflectance data alone using the Kramers-Kronig relation (K-K method) [3]:

$$\frac{\sigma_2}{\sigma_n} = \frac{2}{\pi\omega} \int_{0+}^{\infty} (1 - \frac{\sigma_1}{\sigma_n}) d\omega' + \frac{2\omega}{\pi} \int_{0+}^{\infty} \frac{\sigma_1(\omega')}{\sigma_n(\omega^2 - \omega'^2)} d\omega'.$$

The problem was reduced to the question to find a combination of  $\sigma_1(\omega)$  and  $\sigma_2(\omega)$ , which was consistent with the K-K relations and which gave the measured reflectance. We started with a trial function of  $\sigma_2(\omega)/\sigma_n(\omega) \propto 1/\omega$  (a London approximation). However the combination of  $\sigma_1(\omega)$  and  $\sigma_2(\omega)$  converged rather rapidly in iteration procedures, these conductivities didn't give the good fit to the measured reflectance. These conductivities are shown in Fig. 3 by thin solid curves.

The superconducting gap ( $2\Delta$ ) was estimated at the same value,  $37.6 \text{ cm}^{-1}$ , by both methods. However,  $\sigma_1$  by the K-K method was two times [4] as large as that by the R-T method and  $\sigma_2$  by the K-K method was extremely smaller than that by the R-T method in the high frequency region.

### 4. Conclusion

Although the magnitude of  $2\Delta$  agreed with each other, the conductivity obtained from the solution of two simultaneous equations with measured reflectance and transmittance didn't agree with that by the ordinary method using the Kramers-Kronig relations with the measured reflectance alone in the case of the NbN thin films.

### References

- [1] E. Kawate *et al.* Proc. SPIE **4103** (2000) 30.
- [2] E. Kawate. to be submitted to Appl. Opt.
- [3] P.J.M. van Bentum *et al.* Physica **138B** (1986) 23.
- [4] H.S. Somal *et al.* Phys. Rev. Lett. **76** (1996) 1525.