Magnetic response of hard type-II superconductors with a semicircular indentation

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Abstract

The magnetic response of superconductors with a semicircular indentation in parallel field is studied. The indentation is placed in a long superconductor wedge. The Meissner and vortex fields are calculated for any indentation radius in the London background. This allowed us to obtain the energy and the forces acting on a vortex in a given position inside the superconductor. The entrance magnetic field dependence on the indentation radius is calculated and compared with the entrance field for an indentation free semi-infinite superconductor. The role of the indentation on the surface barrier and on the vortex dynamics is also discussed.

Key words: vortex dynamics;surface barrier;critical current

The vortex dynamics of a type II superconductor is highly dependent on the geometry of its surface, even in the parallel field geometry. For instance, a semi-infinite superconductor with a flat surface presents a barrier against vortex entrance, the so-called Bean-Livingston barrier [1]. Surface roughness can depreciate this barrier, as pointed out by the weaker electromagnetic interaction between vortex lines and defects of the coherence length size on the specimen surface [2]. However, information about the interaction of the vortex lines and defects with size larger than the penetration depth, λ, is not completely understood.

In this work the magnetic response of a wedge shaped superconductor with a circular indentation in the edge (see Fig. 1) is calculated assuming London background. The interaction force between the vortex and the surface (self force) is also calculated for $\beta = \pi$.

The 2-D London equation for an isotropic superconductor

$$-\lambda^2\nabla^2 h(\rho,\phi) + h(\rho,\phi) = \frac{\Phi_0}{\mu_0}\frac{\delta(\rho-\rho_i)}{\rho}\delta(\phi-\phi_i), \quad (1)$$

with the Dirichlet boundary conditions $h(\rho = a, \phi) = h(\rho, \phi = 0) = h(\rho, \phi = \beta) = H_e$, is solved using the Green's function method [3,4]. Above (ρ_i, ϕ_i) is the vortex position, H_e is the external field and the fields are supposed parallel to the specimen surface.

The solution for an arbitrary arrangement of vortices and $H_e = 0$ is shown to be

$$h(\tilde{\rho},\,\phi) = \frac{\Phi_0}{\mu_0\lambda^2}\sum_i h_v(\tilde{\rho}_i, \phi_i),$$

$$h_v(\tilde{\rho}_i, \phi_i) = \frac{2}{\beta}\sum_{m=1}^{\infty} \sin\left(\frac{m\pi\phi}{\beta}\right)\sin\left(\frac{m\pi\phi_i}{\beta}\right) \times$$
$$\times[I_{\alpha_m}(\tilde{\rho}_<) - I_{\alpha_m}(\tilde{a})K_{\alpha_m}(\tilde{\rho}_<)/K_{\alpha_m}(\tilde{a})]K_{\alpha_m}(\tilde{\rho}_>), \quad (2)$$

where $I_n(x)$ and $K_n(x)$ are the modified Bessel functions, $\alpha_m = m\pi/\beta$, the tilded quantities are divided by λ and $\tilde{\rho}_>$ ($\tilde{\rho}_<$) is the larger (smaller) between $\tilde{\rho}$ and $\tilde{\rho}_i$.

In the case $\beta = \pi$ the expression for $h_v(\rho_i, \phi_i)$ can be greatly simplified

$$h_v(\rho_i, \phi_i) = h_1(\rho_i, \phi_i) - h_1(\rho_i, -\phi_i) -$$
$$-\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{I_m(\tilde{a})}{K_m(\tilde{a})}K_m(\tilde{\rho})K_m(\tilde{\rho}_i)\sin(m\phi)\sin(m\phi_i), \quad (3)$$

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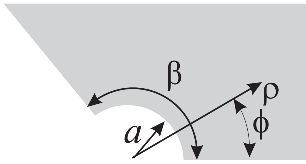

Fig. 1. Semi-infinite superconductor (darker region) with an indentation of radius a in its edge.

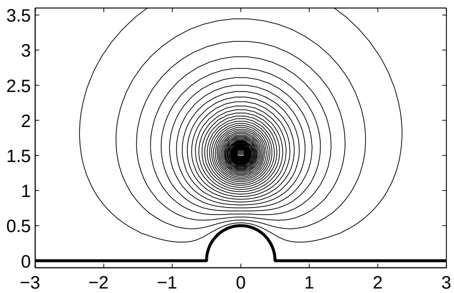

Fig. 2. Current density streamlines (or contour lines of the magnetic field) of one vortex near an indentation of radius $\lambda/2$. The axis are displayed in units of λ.

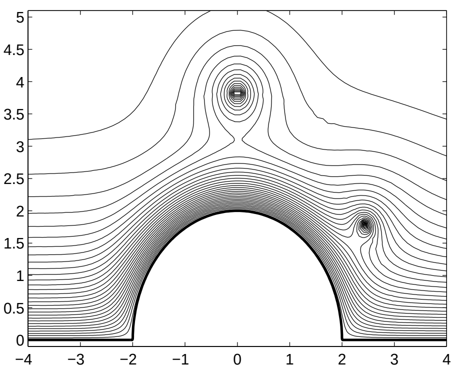

Fig. 3. Current density streamlines for the Meissner response and of two vortices near an indentation of radius 2λ. In this case $H_e = \Phi_0/\mu_0\lambda^2$. The axis are displayed in units of λ.

where

$$h_1(\rho_i, \phi_i) = \frac{1}{2\pi} K_0(\sqrt{\tilde{\rho}^2 + \tilde{\rho}_i^2 - 2\tilde{\rho}\tilde{\rho}_i \cos(\phi - \phi_i)}), \quad (4)$$

is the well known vortex field in an infinite superconductor. This description of the magnetic field by one vortex and one antivortex terms plus another due to the indentation. One recovers the result of one vortex next to a plane surface letting $a \to 0$.

Figs. 2 and 3 show vortices near the indentation. In the first one the current density streamlines (or contour lines of the magnetic field) are shown for an indentation of radius $a = \lambda/2$. In Fig. 3 two vortices (at 1.8λ and λ from the indentation border) and the Meissner response are shown.

From the expression for the current density, obtained

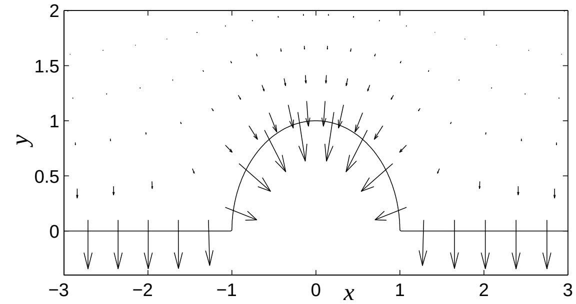

Fig. 4. Vector field plot of the interaction force between one vortex and the surface for an indentation of radius λ. The axis are displayed in units of λ.

from Eqs. 3 and 4, the self force can be calculated [1],

$$\mathbf{f}_s = -\hat{y} K_1(2y_i) + \hat{\phi}\, \frac{2}{\tilde{\rho}} \sum_{m=1}^{\infty} m\, c_m K_m(\tilde{\rho}_i) \sin(2m\phi_i) -$$

$$- \hat{\rho}\, 2 \sum_{m=1}^{\infty} c_m [K_{m+1}(\tilde{\rho}_i) + K_{m-1}(\tilde{\rho}_i)] \sin^2(m\phi_i), \quad (5)$$

where $c_m = I_m(\tilde{a}) K_m(\tilde{\rho}_i)/K_m(\tilde{a})$. In Fig. 4 $\mathbf{f}_s$ is plotted near an indentation. The arrows indicate that the self force is always directed towards the surface and its modulus at a distance ξ from the surface is not much affected by the indentation for this radius. Although not shown here, $\mathbf{f}_s(\rho, \pi/2)$ for $a << \lambda$ is larger compared with the self force felt by a vortex near a flat surface. The detailed vortex dynamics needs also the Meissner force, which pushes the vortex into the superconductor. This work is under way and will be reported elsewhere.

In conclusion we calculated the magnetic fields in a semi-infinite superconductor with an indentation. The vortex magnetic field and self force are calculated analytically, while the Meissner response is obtained numerically. The comparison between the self forces for the flat and indented surfaces show that this force is not very sensitive to the indentation when $a \geq \lambda$.

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