Hanbury Brown Twiss effects in channel mixing normal-superconducting systems

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Abstract

An investigation of the role of the proximity effect in current cross correlations in multiterminal, channel-mixing, normal-superconducting systems is presented. The proposed experiment is an electrical analog of the optical Hanbury Brown Twiss intensity cross correlation experiment. A chaotic quantum dot is connected via quantum point contacts to two normal and one superconducting reservoir. For dominating coupling of the dot to the superconducting reservoir, a magnetic flux of the order of a flux quantum in the dot suppresses the proximity effect and reverses the sign of the cross correlations, from positive to negative. In the opposite limit, for a dominating coupling to the normal reservoirs, the proximity effect is weak and the cross correlation are positive for a nonideal contact between the dot and the superconducting reservoir. We show that in this limit the correlations can be explained with particle counting arguments.

Key words: current correlations; mesoscopic transport; superconducting proximity effect

1. Introduction

In normal mesoscopic multiterminal conductors, the cross correlation between currents flowing into different terminals are manifestly negative. As shown in Ref. [1], this is a consequence of the fermionic statistics of the electrons. Recently, such negative correlations were observed experimentally [2], in an electrical analog [1,3,4] of the Hanbury Brown Twiss experiment [5] with photons.

The manifestly negative cross correlations were derived under the assumption of noninteracting electrons. In contrast, long range Coulomb interactions between electrons can give rise to positive cross correlations between currents at capacitive contacts [6]. Moreover, in Ref. [7] it is demonstrated that for inelastic scattering between edge states in a multiterminal conductor in the quantum Hall regime, the current cross correlation can be positive. Another situation where positive cross

correlations are possible is when the normal conductor is connected to a superconducting reservoir [8].

In a normal-superconducting system, electrons can be converted into holes via Andreev reflection at the interface between the normal conductor and the superconductor. The Andreev reflection thus introduces correlations between electrons and holes in the normal conductor, a phenomena known as the proximity effect [9,10]. The influence of the proximity effect on the current auto-correlations, i.e. the shot noise, in two-terminal normal-superconductor junctions has been studied in Refs. [11,12].

In multiterminal conductors, Andreev reflection can lead to positive cross correlations between currents flowing in the contacts to the normal reservoirs [8,13]. This was originally predicted for single mode conductors [8,13]. In contrast, a discussion of a very asymmetrical geometry in which one of the contacts is a tunneling tip suggested negative correlations in the presence of channel-mixing scattering [14]. Interestingly, in Ref. [15] it was shown that in a multiterminal diffusive normal-superconducting system with perfect in-

terfaces between the normal conductor and superconducting reservoir, the cross correlations are manifestly negative in the absence of the proximity effect. Positive correlations can be enforced with the help of ferromagnetic contacts [16] a case which we will not discuss here further.

Clearly, these very different predictions and their sensitivity to the specific choice of parameters and geometry demand a more systematic investigation of the role of channel-mixing and the proximity effect. To investigate the role of channel mixing Boerlin et al. [17] have analyzed a geometry with diffusive tunnel junctions. Independently, the authors [18] present a systematic analysis of the current correlations in a system consisting of a chaotic quantum dot connected via point contacts to one superconducting and two normal reservoirs. Using the scattering approach for normal-superconducting systems [8] in combination with a random matrix theory [9,19] description of the chaotic dot has the advantage of allowing us to analyze both the case with and without proximity effect on the same footing. The *generic* properties of the model also makes our results qualitatively relevant for multi-terminal normal-superconducting structures with random scattering.
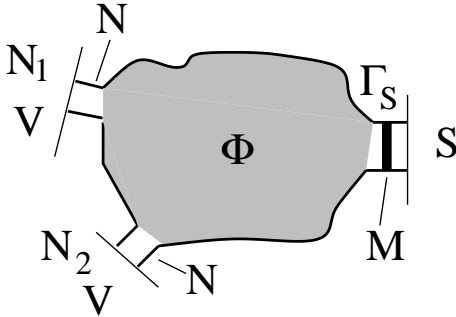
2. The model

Fig. 1. A chaotic quantum dot (grey shaded), acting as a beam-splitter, is connected to two normal reservoirs (N_1 and N_2) and one superconducting reservoir (S) via quantum point contacts. There is a magnetic flux Φ in the dot.

The three terminal dot-superconductor junction is shown in Fig. 1. A chaotic quantum dot (see Ref. [9] for definition) is connected to two normal reservoirs (N_1 and N_2) and one superconducting reservoir (S) via quantum point contacts. The contact to the normal reservoir is perfect, the contact to the superconducting reservoir has a mode independent transparency Γ_S.

The contacts support N and M transverse modes respectively. The conductance of the point contacts are much larger than the conductance quanta $2e^2/h$, i.e.

$N, M\Gamma_S \gg 1$, so Coulomb blockade effects in the dot can be neglected. The two normal reservoirs are held at the same potential V and the potential of the superconducting reservoir is zero. The magnetic flux in the dot is Φ.

3. Current cross correlations

Due to the random scattering in the dot, the current $I_i(t)$ in contact i fluctuates around its quantum statistical average $\bar{I}_i$. We calculate the zero-frequency spectral density of the current cross-correlations

$$P_{12} = 2 \int dt \, \overline{\Delta I_1(t) \Delta I_2(0)}, \qquad (1)$$

where $\Delta I_j(t) = I_i(t) - \bar{I}_i$. The correlation P_{12} can be expressed [8] in terms of the scattering matrix S of the total system, dot and superconductor. To proceed S can be expressed [9] in terms of the scattering matrix S_d of the dot and the Andreev reflection amplitude at the contact-superconductor interface.

We consider the limit of zero temperature and a potential eV much lower than the inverse dwell time of the dot, E_{Th}, where the energy dependence of the scattering matrix S_d can be neglected. We also assume $eV \ll \Delta$, where Δ is the superconducting gap. This restricts the transport to an energy interval where no quasiparticles can escape into the superconductor. In this regime, the scattering matrix S describes only scattering between the normal reservoirs.

Noting that the current fluctuation is just the sum of the fluctuations of electron and hole currents, the noise power can be conveniently written [8],

$$P_{12} = P_{12}^{ee} + P_{12}^{hh} + P_{12}^{eh} + P_{12}^{he} \qquad (2)$$

where $P_{12}^{\alpha\beta}$ is the correlation between α and β quasiparticle currents. Unlike the cross correlations in purely normal systems [1], which are manifestly negative, the cross-correlation P_{12} can be positive, because the correlations between different types of quasiparticles, $P_{12}^{eh} + P_{12}^{he}$, are positive.

The ensemble averaged correlations $\langle P_{12} \rangle$ are calculated using the statistical properties of the scattering matrix S_d of the dot [20]. The details of these calculations are presented in Refs. [18,21], here we just show the result in Fig. 2 for various transparencies Γ_S of the dot-superconductor contact.

Two different regimes for the magnetic flux are considered. For a flux much smaller than the flux quanta, $\Phi \ll h/e$, the magnetic flux has no effect on the proximity effect and can be completely neglected. In the opposite regime, a magnetic flux larger than the flux quantum, $\Phi \gg h/e$, effectively breaks the time rever-

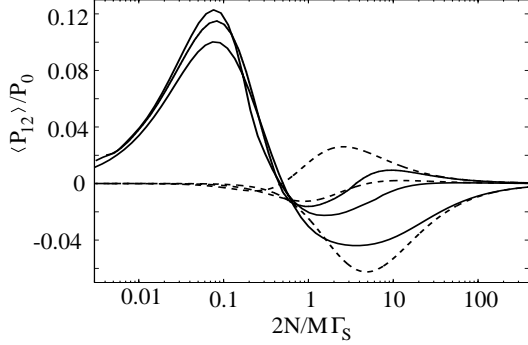sal symmetry in the dot and suppresses the proximity effect [19].

Fig. 2. The ensemble averaged current cross-correlation $\langle P_{12} \rangle$ with (solid) and without (dashed) proximity effect in the dot, as a function of $2N/M\Gamma_S$. The transparency Γ_S is $1.0, 0.8$ and 0.6 from bottom to top (counting at $2N/M\Gamma_S = 10$).

We see from Fig. 2 that suppressing the proximity effect has a very different effect in the two limits; dominating coupling of the dot to the normal reservoirs, $M\Gamma_S \ll N$, and to the superconducting reservoir $M\Gamma_S \gg N$. In the limit of strong coupling to the superconducting reservoir, $M\Gamma_S \gg N$, suppressing the proximity effect with a magnetic flux in the dot completely suppresses the positive correlations.

In this limit, in the presence of the proximity effect, there is a gap in the spectrum around the Fermi energy in the dot [19]. As a consequence, quasiparticles injected from one normal reservoir are Andreev reflected, effectively direct at the contact-dot interface [22], with unity probability back to the same reservoir. This is shown in Fig. 3 for a perfect contact to the superconducting reservoir, $\Gamma_S = 1$, however, the scattering probabilities (as a function of $2N/M\Gamma_S$) are essentially independent of Γ_S. For $2N/M\Gamma_S \to 0$, the scattering process is thus deterministic, there is no partition of incoming quasiparticles and hence no noise, $P_{12} = 0$.

Increasing the coupling to the normal reservoir, the probability of normal reflection as well as cross Andreev reflection, from one reservoir to the other, becomes finite. As is clear from Fig. 3, the normal reflection is the dominant process of the two. On a formal level, we can thus neglect the terms in P_{12} containing the cross Andreev reflection amplitude. As seen in Fig. 3, this gives $\langle P_{12} \rangle = 2P_0(N/M\Gamma_S)$, positive since only terms in $\langle P_{12}^{eh} \rangle + \langle P_{12}^{he} \rangle$ contribute. Here $P_0 = 4e^3/h$.

Suppressing the proximity effect, the gap in the spectrum is suppressed and the scattering processes are strongly modified. This is clearly seen by comparing the two figures in Fig. 3, showing the scattering probabilities for $\Gamma_S = 1$. In the limit of $2N/M \to 0$, an injected electron, which in the presence of the proximity effect was directly back reflected as hole with unity

probability to the same contact, now has a probability $1/4$ each to leave the dot in the following four ways: as an electron into contact 1 or 2 or a hole into contact 1 or 2.

As a consequence, the correlations become negative, i.e. there are no positive correlations in the absence of the proximity effect, similar to what was found for a metallic diffusive system [15]. Fig. 2 demonstrates that this is independent on the transparency Γ_S of the barrier in the dot-superconductor contact.
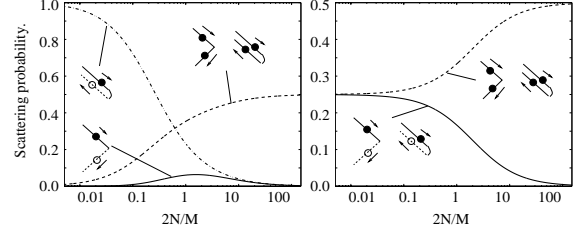

Fig. 3. The probabilities for different scattering processes for an electron incoming from reservoir 1, as a function of $2N/M$ with (left) and without (right) a proximity effect in the dot. The point contact to the superconducting reservoir is perfectly transparent, $\Gamma_S = 1$. Filled (empty) circles denote electrons (holes) and $\searrow$ ($\nearrow$) denotes quasiparticles leaving to reservoir 1(2).

In the opposite limit, with a weak coupling of the dot to the superconductor, $M\Gamma_S \ll N\Gamma_N$, the cross correlations are independent of the proximity effect, as seen in Fig. 2. This is also observed for the different scattering probabilities in Fig. 3. An analytical calculation of the correlations in this limit gives the simple result

$$\frac{\langle P_{12} \rangle}{P_0} = \frac{M}{2N}R_{eh}(1 - 2R_{eh}) \tag{3}$$

where $R_{eh} = \Gamma_S^2/(2 - \Gamma_S)^2$ is the Andreev reflection probability of quasiparticles incident in the dot-superconductor contact. There is a crossover from negative to positive correlations, which occurs for $R_{eh} = 1/2$, i.e $\Gamma_S = 2(\sqrt{2} - 1) \approx 0.83$. In this limit of weak coupling to the superconducting reservoir, where the proximity effect plays no role, it is possible to explain the current correlations with straightforward particle counting arguments.

4. Particle counting model

In the limit of weak coupling to the superconductor, a quasiparticle injected from one of the normal reservoir will at the most scatter once at the contact between the dot and the superconductor (with a probability R_{eh} to Andreev reflect), before leaving the dot. Noting that an Andreev reflection at the dot-superconducting interface effectively leads to an injection of a pair of

particles (electrons or holes), we can equivalently say that in the limit of weak coupling to the superconductor, none of the particles in such an injected pair will ever return to the dot-superconductor contact, but instead directly and independently leave the dot into one of the normal reservoirs, with probability $T_1 = T_2 = 1/2$ for each reservoir. This process is shown schematically in Fig. 4. Based on this, we can, using particle
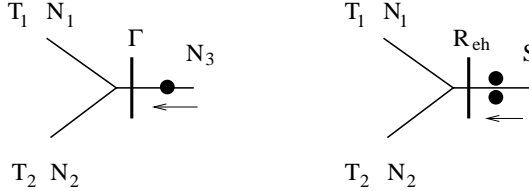

Fig. 4. Left: Injection of single electrons, corresponding to a completely normal system. Right: Injection of pairs of electrons, corresponding to the normal-superconducting system.

counting statistics arguments, calculate the probability $P(N_1, N_2, N_p)$ that $N_1(N_2)$ electrons have ended up in the normal reservoir 1(2), when N_p pairs have tried to enter the dot. From the probability $P(N_1, N_2, N_p)$, we can calculate (the details are presented in Ref. [21]) the cross correlations, giving

$$P_{12} = \langle N_1 N_2 \rangle - \langle N_1 \rangle \langle N_2 \rangle \propto R_{eh}(1 - 2R_{eh}). \qquad (4)$$

This is just the same as was found in Eq. (3), using a completely quantum mechanical approach.

As a comparison, considering the injection of single particles (see Fig. 4), corresponding to the situation when the superconducting reservoir is replaced by a normal reservoir, we instead get manifestly negative cross correlations.

From this we can conclude that in the limit of a weak coupling to the superconductor, the current correlations are determined by the Andreev reflection at the dot-superconductor interface together with independent partitioning of particles inside the dot.

5. Conclusions

In conclusion, we have investigated the relation between current cross correlations and the proximity effect in a three terminal chaotic-dot superconductor junction. We find that both the sign and magnitude of the correlations can be changed by a weak magnetic field in the dot, suppressing the proximity effect. In the limit of weak coupling of the dot to the superconducting reservoir, the current correlation can be found by particle counting arguments.

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