Vortex lattice melting in very anisotropic superconductors influenced by the force-free current

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Abstract

We investigate the renormalization of the elastic matrix (elastic softening) of a vortex array aligned along the c-axis due to the presence of a spatially homogeneous force-free c-axis current. The applied current decreases the stability of the vortex solid phase with respect to thermal fluctuations, shifting the vortex-lattice melting to lower temperatures and/or fields.

Key words: Very anisotropic superconductors; layered superconductors; vortex lattice melting transition; force-free current.

Transport measurements are very useful for studying the vortex lattice melting transition. The magnetic field or temperature dependence of the in-plane [1] or out-of plane resistivity [2] is often measured and the resistivity onset is attributed to the vortex lattice melting transition. However, one can pose the question of how the applied current itself influences the vortex lattice melting transition. For instance, it is well known that a strong enough force-free current can destabilize the vortex lattice even at zero temperature, limiting the longitudinal critical current density [3,4]. It is thus reasonable to expect that a force-free current with density much smaller than the critical value can suppress the stability of the vortex solid for sufficiently large thermal fluctuations, shifting the vortex lattice melting transition to lower magnetic fields. This physical picture is theoretically studied in this paper.

An applied current flowing along straight vortex lines (aligned here along the z-axis) does not affect them, but affects curved vortex lines (e.g., locally tilted due to thermal fluctuations). The Lorentz force acting on a vortex segment is $\mathbf{f}_l = (\Phi_0 J/c)[\mathbf{e}_z \times \partial\mathbf{u}/\partial z]$, with force-free current density $\mathbf{J} = J\mathbf{e}_z$, flux quantum Φ_0, speed of light c, unit vector $\mathbf{e}_z$ along the z-axis, and displacement $\mathbf{u}$ of the vortex segment from its equilibrium position on a triangular lattice. Using this expression, the elastic free energy $\mathcal{F}_{el}$ in the presence of the force-free current can be written as [4]:

$$\mathcal{F}_{el} = \frac{1}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\Phi_{\alpha\beta}(\mathbf{k})u_\alpha(\mathbf{k})u_\beta(-\mathbf{k}) - \frac{BJ}{c}\int \frac{d^3\mathbf{k}}{(2\pi)^3} ik_z\big(u_x(\mathbf{k})u_y(-\mathbf{k}) - u_x(-\mathbf{k})u_y(\mathbf{k})\big), \quad (1)$$

where B is the magnetic induction along the z-axis and $\Phi_{\alpha\beta}(\mathbf{k})$ is the elastic matrix (see for instance [5]). The integration in (1) should be performed over the first Brillouin zone of the reciprocal lattice: $k_x^2 + k_y^2 < 4\pi B/\Phi_0$, $|k_z| < k_z^{\max}$. For 3D anisotropic superconductors, the maximum value $k_z^{\max}$ of the z-component wave vector is about $1/\xi_c$, with the out-of-plane coherence length ξ_c; while for layered superconductors $k_z^{\max}$ is restricted by the interlayer distance s as $k_z^{\max} \sim 1/s$.

To analyze how the z-axis current influences the vortex lattice melting transition, we can use the Lindemann criterion $c_L^2 a_p^2 = \langle u^2 \rangle$, with Lindemann num-

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ber $c_L \approx 0.1$–0.2 and the intervortex distance $a_p \approx \sqrt{\Phi_0/B}$. The mean square vortex displacement $\langle u^2 \rangle$ produced by thermal fluctuations is defined via the path integral

$$\langle (u(\mathbf{r}_0))^2 \rangle = \frac{1}{Z} \int \mathcal{D}u(\mathbf{r}) \left(u(\mathbf{r}_0)\right)^2 \exp\left(-\mathcal{F}_{el}/T\right) \quad (2)$$

with the partition function Z

$$Z = \int \mathcal{D}u(\mathbf{r}) \exp\left(-\mathcal{F}_{el}/T\right). \quad (3)$$

Because of the diagonal form of the free energy (1) in the $\mathbf{k}$-representation, the mean square displacement can be calculated exactly:

$$\langle u^2 \rangle = T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\Phi_{xx} + \Phi_{yy}}{\Phi_{xx}\Phi_{yy} - \Phi_{xy}^2 - B^2 J^2 k_z^2/c^2}. \quad (4)$$

Next, the integral (4) has to be estimated. In general, the elastic matrix can be expressed as: $\Phi_{xx} = c_{11}k_x^2 + c_{66}k_y^2 + c_{44}k_z^2 + \alpha_L(\mathbf{k})$; $\Phi_{yy} = c_{66}k_x^2 + c_{11}k_y^2 + c_{44}k_z^2 + \alpha_L(\mathbf{k})$; $\Phi_{xy} = \Phi_{yx} = (c_{11} - c_{66})k_x k_y$; with compression $c_{11}(\mathbf{k})$, shear c_{66} and tilt $c_{44}(\mathbf{k})$ $\mathbf{k}$-dependent elastic moduli, and the Labusch parameter α associated with pinning. The complicated $\mathbf{k}$-dependence of the elastic matrix does not allow to estimate the mean square displacement analytically, restricting us to numerical calculations. However, for strongly anisotropic layered superconductors placed in a low c-axis magnetic field $H < \Phi_0/\lambda_{ab}^2$ (λ_{ab} is the in-plane penetration depth), the main contribution to the elastic response comes from the electromagnetic interaction of pancake vortices. In this case we can roughly approximate the component of the elastic matrix as

$$\Phi_{xx} \approx \Phi_{yy} \approx B\overline{U}_{44}, \quad \Phi_{xy} \approx 0, \quad (5)$$

where

$$\overline{U}_{44} = \frac{\Phi_0}{32\pi^2\lambda_{ab}^4} \ln(1 + 4\lambda_{ab}^2/c_L^2 a_p^2) \quad (6)$$

is the $\mathbf{k}$-independent tilt stiffness of a pancake vortex stack. Such an approximation [6] is valid for $|k_z| < k_z^* = \min\left(1/s,\, \gamma\sqrt{\ln(1 + 4\lambda_{ab}^2/c_L^2 a_0^2)}/\lambda_{ab}\right)$. Performing integration in (4) in the region $|k_z| < k_z^*$ and now using the Lindemann criterion, we obtain the melting field:

$$B^{\mathrm{melt}} \approx \pi c_L^2 \Phi_0^2 J \left\{ cT \ln\left(\frac{1 + Jk_z^*/c\overline{U}_{44}}{1 - Jk_z^*/c\overline{U}_{44}} \right) \right\}^{-1}. \quad (7)$$

In the limit of zero current, this expression reproduces a previous result [6]:

$$B^{\mathrm{melt}} = \frac{c_L^2 \Phi_0^3 \sqrt{\ln(1 + 4\lambda_{ab}^2/c_L^2 a_p^2)}}{64\pi\gamma T \lambda_{ab}^3}. \quad (8)$$

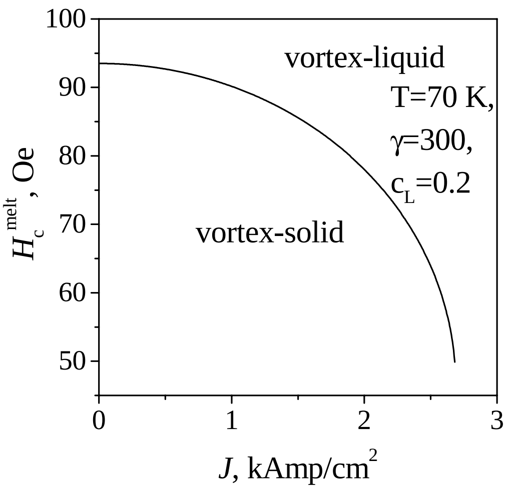

Fig. 1. The calculated H_c versus J phase diagram of the vortex lattice melting transition using parameters representative for $Bi_2Sr_2CaCu_2O_{8+\delta}$ such as $\gamma = 300$, $c_L = 0.2$, $\lambda_{ab} = 2000/\sqrt{1 - T^2/T_c^2}$ Å, s =15 Å, T =70 K, T_c =90 K.

Figure 1 shows the H_c versus J phase diagram using parameters representative for layered superconductors like $Bi_2Sr_2CaCu_2O_{8+\delta}$.

In conclusion, we study the stability of the vortex solid phase under the influence of an applied homogeneous force-free current. It was found that the current suppresses the vortex solid phase, shifting the vortex lattice melting phase diagram to lower magnetic fields. We thus obtain the H_c versus J low-field phase diagram showing the vortex solid and liquid phases.

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