

# Critical magnetic fluctuations induced superconductivity and residual density of states in $CeRhIn_5$ superconductor

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## Abstract

We propose the multiband extension of the spin-fermion model to address the superconducting d-wave pairing due to magnetic interaction near critical point. By solving the unrestricted gap equation with a general d-wave symmetry gap, we find that divergent magnetic correlation length  $\xi$  leads to the very unharmonic shape of the gap function with shallow gap regions near nodes. This unharmonic gap is extremely sensitive to the small amount of disorder and we propose that we can understand the large  $N_{res}(0) = \lim_{T \rightarrow 0} C_p(T)/T$  value and its pressure dependence of the recently discovered  $CeRhIn_5$  superconductor under pressure within this approach.

*Key words:* superconductivity; quantum criticality; CeMIn5; heavy fermions

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## 1. Introduction

Recent discovery of superconductivity in  $CeMIn_5$  ( $M = Co, Rh, Ir$ )[1] has spurred a renewed interest in heavy fermion systems and about the nature of its superconductivity. Motivated by  $CeRhIn_5$  experiment[2], in this paper we examined possible conditions for a d-wave superconductors (1) to create a substantial density of states (DOS) in superconducting phase with a small impurity concentration, and (2) to have a large variation of it as a function of pressure while keeping the constant  $T_c$ [2].

To address these questions we propose a multiband generalization of the spin-fermion model where localized Ce spins  $\mathbf{S}$  are interacting with the conduction electrons (predominantly d band of  $In$ ) via the Kondo exchange coupling  $J$ . In mixed momentum and real space representation the Hamiltonian is written as

$$H = \sum_{\mathbf{k}, \alpha} c_{\alpha}^{\dagger}(\mathbf{k}) \varepsilon(\mathbf{k}) c_{\alpha}(\mathbf{k}) + \sum_{\mathbf{r}, \alpha, \beta} J \mathbf{S}(\mathbf{r}) \cdot c_{\alpha}^{\dagger}(\mathbf{r}) \sigma_{\alpha \beta} c_{\beta}(\mathbf{r}) + H_S \quad (1)$$

where the first term is the kinetic energy and the second describes the Kondo exchange between Ce spins and conduction electron spin density. The last term represents an effective low energy Hamiltonian for the localized spins. The dynamics of the localized spins without long range AFM order coupled with conduction electrons is well captured by the spin correlation function[3],  $\chi(\mathbf{q}, \omega) = \frac{V_0}{i\omega/\omega_0 + \xi^2(\mathbf{q} - \mathbf{Q})^2 + 1}$ , where  $\omega_0$  is a spin relaxation energy scale,  $\mathbf{Q}$  is the 2-dimensional antiferromagnetic vector, and  $\xi$  is the magnetic correlation length. And the weak coupling pairing potential is proportional to the static limit of  $\chi(\mathbf{q}, \omega = 0)$ , and we model it in 2-dimensional FS with  $\xi^{-1} \sim b$  as follows.

$$V(\phi - \phi') = V_d(b) \frac{b^2}{(\phi - \phi' \pm \pi/2)^2 + b^2}. \quad (2)$$

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## 2. Formalism

For simplicity, we assume a circular FS in two dimensions and the effect of the impurity scattering in SC is included with T-matrix approximation. For particle-hole symmetric case  $T_3 = 0$ , and for d-wave OP with isotropic scattering  $T_1 = 0$  (also without loss of generality we can choose  $T_2 = 0$  by U(1) symmetry). Then the self energy is given by  $\Sigma_0 = \Gamma T_0$ , where  $\Gamma = n_i/\pi N_0$ ,  $N_0$  the normal DOS at the Fermi energy,  $n_i$  the impurity concentration. Scattering strength parameter  $c$  is related with the s-wave phase shift  $\delta$  as  $c = \cot(\delta)$ . Now  $T_0(\omega_n) = \frac{g_0(\omega_n)}{[c^2 - g_0^2(\omega_n)]}$ , where  $g_0(\omega_n) = \frac{1}{\pi N_0} \sum_k \frac{i\omega_n}{\tilde{\omega}_n^2 + \epsilon_k^2 + \Delta^2(k)}$ ,  $\tilde{\omega}_n = \omega_n + \Sigma_0$ . With this  $T_0(\omega)$  the following gap equation is solved self-consistently.

$$\Delta(\phi) = -N_0 \int \frac{d\phi'}{2\pi} V(\phi - \phi') \cdot F(\phi') \cdot T \sum_{\omega_n - \omega_D}^{\omega_D} \int d\epsilon \frac{\Delta(\phi')}{\tilde{\omega}_n^2 + \epsilon^2 + \Delta^2(\phi')} \quad (3)$$

Finally, we introduce the FS weighting function  $F(\phi) = \cos^\beta(2\phi)$  ( $\beta = 8$  for all our calculations) to correct the artifact of the circular FS and to mimic the important aspect of a real FS.

## 3. Results

In all calculations we set  $\omega_D = 1$ . Fig.1(a) shows the normalized pairing potentials  $V(\phi)/V_d(b)$  as a function of  $b$  for illustration, and Fig.1(b) shows the solutions of  $\Delta(\phi)$  for the Born limit scatterer ( $c = 1, \Gamma = 0.025$ ). Fig.2(a) shows  $N(\omega = 0)/N_0$  as a function of  $b$  ( $\sim P$ ) for both impurity cases. The result shows that the Born limit scatterer has a stronger dependence on pressure compared to the unitary scatterer. This is because of the opposite trend of  $\gamma = \text{Im}\Sigma(0)$  in unitary scatterer due to the resonant pole. In comparison to the experimental data of  $N_{\text{res}}(0)$ [2](shown in Fig.2(b)), the Born scatterer fits the data better.

In summary, in this paper, we proposed a multiband spin-fermion model as a description of the pairing in *CeMIn*<sub>5</sub> materials. Assuming that the magnetic correlation length  $\xi$  decreases with pressure, we accordingly model the weak coupling pairing potential. Using this potential we showed that the slope of the gap near nodes can be sharply changed. This strong change of the slope can explain the pressure dependence of  $N_{\text{res}}(0)/N_0$  as well as the large value of it in *CeRhIn*<sub>5</sub> superconductor[2] close to the quantum critical limit of  $\xi \rightarrow \infty$  with a small amount of impurities.

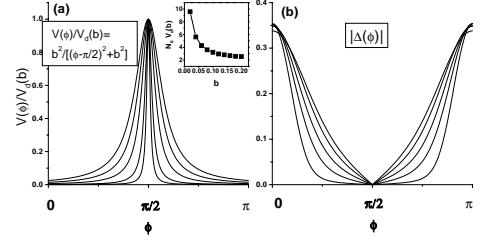


Fig. 1. (a) The normalized pairing potential  $V(\phi)/V_d(b)$  as a function of the exchange momentum  $\phi$ , for different values of  $b$  ( $\sim \xi^{-1}$ ). In increasing order of potential width,  $b = 0.02, 0.08, \dots, 0.2$ .  $\phi = \pi/2$  is the AFM peak momentum  $\mathbf{Q}$ . Inset is the  $V_d(b)$  which is numerically determined to make  $T_c$  constant. (b) The OP solutions  $|\Delta(\phi)|$  for pairing potentials shown in (a) with impurities (Born limit  $c = 1$  and  $\Gamma = 0.025$ ) for all cases. In decreasing order of potential width,  $\Delta(\phi)$  becomes flatter near node.

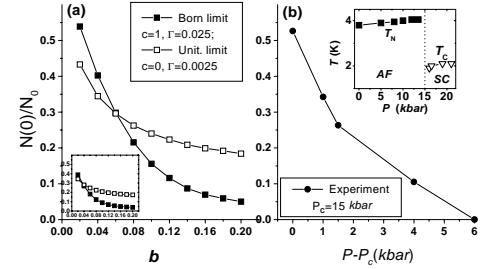


Fig. 2. (a) The normalized DOS  $N(\omega = 0)/N_0$  as a function of  $b$  ( $\sim \xi^{-1} \sim P - P_c$ ) for Born limit scatterer ( $c = 1, \Gamma = 0.025$ , solid square) and for the unitary scatterer ( $c = 0, \Gamma = 0.0025$ , open square). Inset is with  $\beta = 4$  ( $c = 1, \Gamma = 0.03$ , solid square;  $c = 0, \Gamma = 0.003$ , open square); (b) Experimental data from Ref[2]. Inset: the experimental phase diagram of *CeRhIn*<sub>5</sub> (Ref[2]).

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## References

- [1] H. Hegger et al., Phys. Rev. Lett. **84**, 4986 (2000); C. Petrovic et al., Europhys. Lett. **53**, 354 (2001).
- [2] R. A. Fisher et al., cond-mat/0109221.
- [3] A.J. Millis, H. Monien, and D. Pines, Phys. Rev. B **42**, 167 (1990); P. Monthoux, A. V. Balatsky, and D. Pines, Phys. Rev. Lett. **67**, 3448 (1991).