Quantum Hall stripes in a periodic potential.

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Abstract

We consider the influence of a weak external periodic potential on the stripes at half-integer filling fractions of the upper Landau levels. We find the ground state by minimizing the cohesive energy for the stripes aligned perpendicular to the direction of the applied external modulation.

Key words: Quantum Hall effect; stripe phase

1. Introduction

Experimental discovery of the guiding center charge density wave states in partially filled higher Landau levels [1] was anticipated theoretically by Fogler *et. al* [2] by several years. The observations of anisotropic conductivity between filling fractions ν of $9/2$ and $21/2$ is now considered to be convincing experimental evidence for the existence of these striped states at these filling fractions.

It seems natural to study the interplay of interaction-induced periodicity with an external commensurate (or incommensurate) periodic potential. Indeed measurements of the magnetoresistance in lateral superlattices with period close to the expected period of the CDW have been performed [6]. These experiments show a minimum with peaks on both sides when the direction of the current is parallel to the direction of the external modulation and a peak when the direction of the current is perpendicular to it.

In this paper we investigate the structure of the ground state in the presence of a weak periodic potential when the modulation is perpendicular to the direction of the stripes. Minimizing the cohesive energy we obtain modifications of positions and widths of the stripes due to the external potential.

2. Higher Landau level electrons in a periodic potential.

We consider a clean fully spin polarized two-dimensional electron system in a magnetic field and a weak periodic potential (amplitude V_0) with the Hamiltonian:

$$H_{\mathrm{eff}} = \frac{1}{2L_x L_y}\sum_{\mathbf{q}} \rho(\mathbf{q})u(\mathbf{q})\rho(-\mathbf{q}) + + \sum_{\mathbf{q}} \frac{V_0 F(q)}{2}\rho(\mathbf{q})\delta(\mathbf{q}\pm\mathbf{Q}), \quad (1)$$

where $u(\mathbf{q})$ is the screened Coulomb potential [3,2], $\rho(\mathbf{q})$ is the density operator projected on the upper Landau level, and $F(q)$ is the form-factor.

We first recall that [2] after the Hartree-Fock decoupling, the screened Coulomb potential for the uniform in the $\hat{y}$ direction guiding center order parameter, can be approximated by [2]: $u^{eff}_{\mathrm{HF}}(x) = u_0\Theta(2R_c - |x|)$, with $u_0 \equiv \frac{\hbar\omega_c}{2\pi^2 R_c}$, where R_c and ω_c are the cyclotron radius and frequency respectively (the contact part of the potential that is unimportant for our discussion has been omitted).

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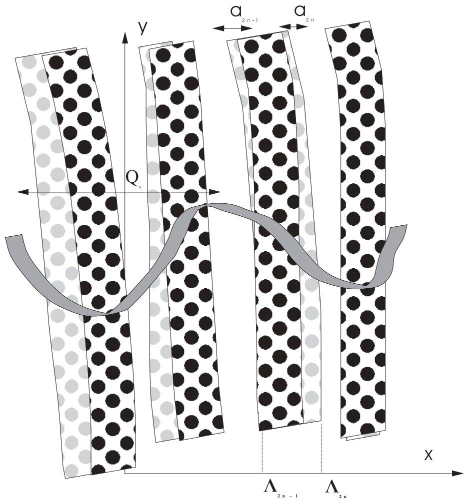

Fig. 1. Geometry of stripes adjusting to the potential. The modified stripes are shown in black. For comparison the uniform stripes are shown in grey. Left edge of the n^{th} stripe shifts by a_{2n}, right edge - by a_{2n+1}. The applied external modulation is in the $\hat{x}$ direction.

For $V_0 = 0$, the ground state has been found to consist of stripes with the period $\Lambda = 2\sqrt{2}R_c$ uniform in $\hat{y}$ direction. That is electron states with the guiding centers X are filled for $(n - 1/4)\Lambda < X < (n + 1/4)\Lambda$, where n is an integer labelling the stripe.

The cohesive energy of such stripe state has been found to be: $E^0_{\rm coh} = (-3 + 2\sqrt{2})u_0 R_c$.

In this contribution we consider the potential with the direction of the modulation perpendicular to the stripes ($\mathbf{Q} = (Q_x, 0)$). In this case each stripe can expand or shrink as well as move as a whole with respect to their unperturbed positions to take advantage of the external potential.

3. Stripes Aligned Perpendicular to the Direction of Modulation.

With weak, but nonzero external potential strength, we expect the stripe width and position to change as the stripes adjust to the potential. To take into account both the change in the stripe position and width we introduce the following parameterization for left and right edges of the n^{th} stripe respectively: $(n - \frac{1}{4})\Lambda + a_{2n}$ and $(n + \frac{1}{4})\Lambda + a_{2n+1}$. This parameterization is illustrated on Fig. 1.

We calculate the cohesive energy of the perturbed stripes and look for positions of modified stripes for arbitrary values of the cohesive energy $E_{\rm coh}$. We then find the minimal value of the cohesive energy when solutions of the assumed form are possible. To this end we go to the continuum limit, i.e. promote n to the status of a continuous variable x and introduce $f(x) = a_{2n}$. This results in a quadratic in $f'(x)$ equation with a nonlinear term. Since corrections to the initial stripe positions a_{2n} (and, consequently, $f(x)$) will be seen to be proportional to $\sqrt{V_0 F(Q_x)/u_0}$, the nonlinear terms can be expanded.

The existence of real solutions to the linearized equation determines the minimal value of $E_{\rm coh}$ when such a solution is allowed. To the second order in $V_0 F(Q_x)/u_0$ this value is

$$E_{\rm coh} = E^0_{\rm coh} + \frac{V_0 F(Q_x)}{Q_x \Lambda}\left(2\sin\frac{Q_x\Lambda}{4} - \sin\frac{Q_x\Lambda}{2}\right) - \\ -\frac{(V_0 F(Q_x))^2}{8u_0\Lambda}\left(\cos\frac{Q_x\Lambda}{4} - \frac{4}{Q_x\Lambda}\sin\frac{Q_x\Lambda}{4}\right)^2 \qquad (2)$$

First, an interaction independent, term reflects the cost of arranging free electrons in stripes in the external field. The second term is the energy gain due to adjustment of stripes to the external potential and is quadratic in the amplitude of the potential. In the long wavelength limit ($Q_x\Lambda \ll 1$) the value of the cohesive energy takes on the form: $E_{\rm coh} = E^0_{\rm coh} - (V_0 F(Q_x))^2/72u_0\Lambda\left(\frac{Q_x\Lambda}{4}\right)^4$ in agreement with the long wavelength theory.

Having determined the minimal cohesive energy, we construct solutions for positions of the modified stripes. These solutions describe modulations of the edges' positions with the amplitude of $l_0 = \frac{2}{Q_x\Lambda}\sqrt{|\frac{8V_0F(Q_x)}{u_0 Q_x}\sin\frac{Q_x\Lambda}{4}|}$. It is worth noticing that the amplitude of modulations is nonanalytic in v. When the external period equal to an integer (commensurate periods) and half-integer multiples of the natural period Λ, these modulations are periodic.

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