

Heterogeneous High Temperature Cuprate Superconductors

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Abstract

The boundary between highly under-doped and over-doped cuprate superconductors is studied within the framework of the slave-boson mean-field theory of the t - J model. Because of the proximity effect of singlet resonating valence bond order, the superconductivity appears at the boundary even when there is no bulk superconductivity. Remarkably, the transition temperature of this “boundary superconductivity” exceeds the maximum superconducting transition temperature of the base material in the bulk.

Key words: high temperature superconductivity; t - J model; boundary

1. Introduction

In the slave-boson mean-field theory of the t - J model, the singlet resonating valence bond (s-RVB) order of spinons and the bose condensation of holons are considered to be essential ingredients of the superconductivity [1–4]. Until now, it has been clarified that this picture explains the most physical properties of the cuprate superconductors very well [5,6]. Although there still remain some subtle questions about the validity of the spin-charge separation, the t - J model may be considered as an effective model, which captures the essence of the cuprate superconductors.

In this paper, we consider the situation where the doping rate is spatially varying. We especially study the boundary between highly over-doped and under-doped regions, existing in the same CuO_2 plane. We consider that, in the over-doped region, holons are bose-condensed and, in the under-doped region, the s-RVB order exists. In general, when an ordered phase touches a disordered phase at a boundary, the order penetrates into the disordered region with a characteristic penetration length, a phenomenon known as the “proximity effect”. Therefore, in our case, it is

expected that the holon condensate and the s-RVB order coexist at the boundary, suggesting a possibility of finding superconductivity there. Remarkably, the transition temperature of this “boundary superconductivity” can be as high as the transition temperature of the s-RVB order, namely the pseudogap temperature, which largely exceeds the maximum superconducting transition temperature of the base material.

Our idea has some relation with the “spin-gap proximity effect” previously introduced by Emery, Kivelson and Zacher [7] based on a somewhat different model of high temperature superconductors. Here we especially study, based on a simple Ginzburg-Landau (GL) theory, the linear boundary between over-doped and under-doped region existing in a two-dimensional CuO_2 plane. This situation may also be accessible employing several experimental techniques.

2. Ginzburg-Landau theory of t - J model

The Hamiltonian of the t - J model is given by

$$H = -t \sum_{\langle i,j \rangle \sigma} (f_{i\sigma}^\dagger f_{j\sigma} b_j^\dagger b_i + \text{h.c.}) + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_i \lambda_i \left(\sum_{\sigma} f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i - 1 \right), \quad (1)$$

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where $f_{i\sigma}$ and b_i are the annihilation operators of the spinon and holon, respectively, with the index i and σ standing for the lattice site in two dimension and the spin index, respectively. \mathbf{S}_i denotes $\frac{1}{2} \sum_{\alpha\beta} f_{i\alpha}^\dagger \sigma_{\alpha\beta} f_{i\beta}$ with $\sigma_{\alpha\beta}$ being Pauli matrices. λ_i is the Lagrange multiplier to exclude double occupancy. Within the framework of the slave-boson mean-field theory [3,4], two order parameters (OP's) are introduced to describe the low temperature phase of the cuprate superconductors, which are the s-RVB OP, $\Delta_{ij} = \frac{3J}{8} \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle$, and the holon condensate density, $B_i = \langle b_i \rangle$.

Our next purpose is to derive the GL theory which describe the spatial variation of Δ_{ij} and B_i . We assume

- 1) B_i is almost fixed to the density of dopants because of a strong Coulomb interaction. Therefore it is treated as an external parameter.
- 2) Δ_{ij} favors d -wave symmetry and it is described by a single complex OP, $\Delta_d(\mathbf{r})$, in the continuum limit.

The GL free energy of $\Delta_d(\mathbf{r})$ is obtained using a conventional perturbation technique as [8]

$$F_{\text{s-RVB}} = \alpha_d |\Delta_d|^2 + \beta |\Delta_d|^4 + \gamma_d |\Pi \Delta_d|^2, \quad (2)$$

where $\Pi = -i\nabla + (2\pi/\phi_0)\mathbf{A}$ with $\phi_0 = hc/(2e)$. $\mathbf{A}(\mathbf{r})$ is the vector potential. Here the CuO_2 plane is set parallel to x - y plane. The GL coefficients are given by

$$\alpha_d = 2N(0)(1-\delta)^2 \ln \frac{T}{T_d}, \quad (3)$$

$$\beta = \frac{21\zeta(3)N(0)}{2\pi^2 T^2} (1-\delta)^4 \equiv \frac{c_1}{T^2 D}, \quad (4)$$

$$\gamma_d = \frac{7\zeta(3)N(0)D^2}{16\pi^2 T^2} \equiv \frac{c_2 D}{T^2}, \quad (5)$$

where $N(0)$, δ and D are the density of states at Fermi energy, the doping rate and the band width, respectively. $\zeta(x)$ is the zeta function. The transition temperature is given by $T_d \simeq T^* \exp[-1/\{2VN(0)(1-\delta)^2\}]$ with $T^* = 2De^\gamma/\pi$ where γ is the Euler's constant and V denotes $3J/8$. The doping dependence of the GL coefficients comes from T_d , α_d and β . Among them, the dependence arising from T_d is dominant.

3. The boundary superconductivity

We assume that the holon condensate and s-RVB order occupy the region $x > 0$ and $x < 0$, respectively. This situation is incorporated by setting $T_d \simeq T$ at $x < 0$ and $T_d \ll T$ at $x > 0$. We denote the transition temperature of the latter region by T'_d . The solution of $\Delta_d(\mathbf{r})$, then, reads

$$\Delta_d = \bar{\Delta}_d \left\{ 1 - \frac{\xi_F e^{x/\xi_F}}{\xi_F + \xi'_F} \right\} \quad \text{for } x < 0, \quad (6)$$

$$\Delta_d = \bar{\Delta}_d \frac{\xi'_F e^{-x/\xi'_F}}{\xi_F + \xi'_F} \quad \text{for } x > 0, \quad (7)$$

where $\bar{\Delta}_d$ is the equilibrium value of the gap at $x \rightarrow -\infty$. We note that Δ_d and its derivative is continuous at $x = 0$. The coherence lengths, ξ_F and ξ'_F , characterize the strength of the proximity effect. These are given by

$$\xi_F \simeq \sqrt{\frac{c_2}{2}} \frac{D}{T_d} \frac{1}{\sqrt{1-T/T_d}}, \quad \xi'_F = \frac{\sqrt{c_2}}{\sqrt{\ln T/T'_d}} \frac{D}{T}.$$

c_1 and c_2 are constants defined in eq. (5). Since the holon condensate is assumed in $x > 0$, we conclude that the superconductivity appears in the region $0 < x < \xi'_d$.

4. Discussion

We have demonstrated that the superconductivity appears at the boundary due to the proximity effect of s-RVB order. Except when ξ_d is divergently large (namely $T \sim T_d$), the energy gap is of the order of $\bar{\Delta}_d$ and the width of the superconducting region is of the order of the bulk superconducting coherence length of the base material at $T = 0$. Therefore it may be experimentally observable. For example, the field effect transistor may be an appropriate experimental setup to realize our proposal [9], if the spatially varying doping rate can be managed. Careful crystal growth may also realize this system.

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