Scalable quantum computing using persistent current qubits with Josephson junctions

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Abstract

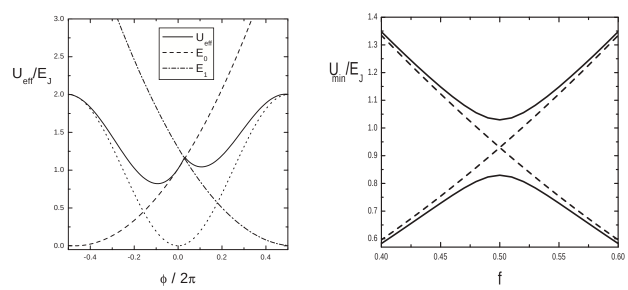
We study quantum logic gates using two persistent current states as a qubit in the superconducting ring with Josephson junction. We use an rf SQUID as a single persistent current qubit. An effective double-well potential, where quantum tunneling is possible, exists in the single qubit. The degeneracy of the two persistent current states is lifted by tunneling. The coupling between qubits is performed by the inductive current connecting qubits. A scalable qubit is constructed by making quantum circuit connecting many qubits in a topologically the same way. Switching each qubit is possible by inserting dc SQUID into the line coming out of qubit.

Key words: SQUID; switching; scalable qubit

Superconducting phase qubits have the advantage of relatively long dechorence time and scalability[1]. Recently the quantum superposition of the phase states was observed[2,3]. However the two qubit operation is not yet achieved. Here we suggest the scheme of the controlled-NOT gate using the current in the loop connecting the two qubits and the scalable design of the quantum computing circuit.

We consider an rf-SQUID as a basic unit of qubit. The total energy E is written as $E = E_C + E_n + E_J(1-\cos\phi)$, where $E_C = \frac{1}{2}\left(\frac{\Phi_s}{2\pi}\right)^2 C\dot{\phi}^2$ is the charging energy and E_n is the sum of the kinetic and the induced energy[4], i.e., $E_n = E_k + E_{ind}$, where $E_k = \frac{\hbar^2 k_n^2}{2m_c}N_c$ and $E_{ind} = (1/2)LI^2 = \frac{1}{2L}(\Phi_t - \Phi_{ext})^2$. Here, $m_c = 2m_e$ is the mass of the Cooper pair, N_c the number of Cooper pairs, and Φ_t and Φ_{ext} are total and external fluxes piercing into the loop of the SQUID, respectively.

The expression for the wave vector at a quantum state n is given by the periodic boundary condition[4] for the phase of the wave function of the SQUID loop, such that $k_n l = -2\pi\Phi_t/\Phi_s + 2\pi n - \phi$, where l is the circumference of the loop, and ϕ the phase difference across the Josephson junction. Combining this boundary condition with the total flux expression, $\Phi_t = \Phi_{ext} + \Phi_{ind}$, where $\Phi_{ind} = LI$, the wave vector is written as $k_n = 2\pi(n - f - \frac{\phi}{2\pi})/(1+\gamma)l$, where $\gamma = \frac{q^2 N_c}{l^2 m_c}L$ is the dimensionless reduced inductance of the loop and $f \equiv \frac{\Phi_{ext}}{\Phi_s}$. The current, on the other hand, is given by



(a) (b)

Fig. 1. (a) A plot for the lowest energy giving effective potential in units of E_J for f=0.47 as a function of Josephson phase. (b) The energy spectrum of the superposed states.

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$I_n = \frac{n_c A q}{m_c}\hbar k_n$, where $n_c = N_c/Al$ is the Cooper pair density, A the cross section, and $q = 2e$ the Cooper pair charge. Thus the sum E_n is expressed by $E_n = \epsilon_o(n - f - \frac{\phi}{2\pi})^2/(1+\gamma)$, where $\epsilon_o = 2\pi^2\hbar^2 n_c A/m_c l$.

The double-well structure shown in Fig. 1 (a) is given by taking the lowest energy for the quantum states $n = 0$ and 1 for the effective potential $U_{eff}(\phi) = E_n + E_J(1 - \cos\phi)$. The double well structure with a cusp at the central part is different from other structures that also use persistent currents as a qubit. The tunnelling between these two states results in the energy spectrum shown in Fig. 1 (b).

The controlled-NOT gate can generate the universal quantum logic gate with the single qubit rotation. To implement a controlled-NOT gate, we consider a two qubit coupling (Fig. 2). The energy corresponding to the minima of the effective potential of the system composed of a qubit pair and connecting loop is obtained numerically (Fig. 2 (c)). Two qubits of Fig. 2(a) are in the same state and have relatively large current flow in the connecting loop. The energy in this case is higher than that of Fig. 2 (b) where two qubits are in different states. The Hamiltonian of the coupled qubit can be written in spin-1/2 notation as $H_{coup} = E(\sigma_{1z} + \sigma_{2z}) + J\sigma_{1z}\sigma_{2z} + \text{const.}$, where J depends on the distance between the two qubits. A resonant pulse applied on the second qubit can transform the state $|\downarrow\downarrow\rangle(|\downarrow\uparrow\rangle)$ into $|\downarrow\uparrow\rangle(\downarrow\downarrow\rangle)$. However the state $|\uparrow\downarrow\rangle(\uparrow\uparrow\rangle)$ does not respond to this pulse and remains in the same state. This discriminated operation makes it possible to perform the controlled-NOT operation[5].

A possible design to achieve a scalable computing and the selective coupling between qubits is presented in Fig. 3 (a). The switches make it possible to couple any selected pair of qubits. We set the cross section of the loop of the switching SQUID as $A_x = A'/2$. Then the current conservation gives $k' = (k_{x1} + k_{x2})/2$. The current flowing in the connecting loop can be obtained by numerically calculating k' of Fig. 3 (b) using above conservation relation and the periodic boundary conditions for the connecting loop and for the switching SQUID, such that $k_{x1}\frac{l_x}{2} + \phi_{x1} + k'l' + \tilde{\phi} = 2\pi p$ and $-k_{x1}\frac{l_x}{2} + k_{x2}\frac{l_x}{2} - \phi_{x1} + \phi_{x2} = -2\pi\frac{\Phi_x}{\Phi_s}$, where $\tilde{\phi}$ is the sum of phase evolutions at the Josephson junctions in the qubits and the other switch, p is an integer and l', l_x are the circumferences of the connecting loop and the switch, respectively. The phases ϕ_{x1} and ϕ_{x2} can be deleted using the current conservation relations for the connecting loop. One can easily obtain these conservation relations: $-n_c\frac{A'}{2}\frac{k_{xi}}{m_c} + E_{Jx}\sin\phi_{xi} = 0$ $(i = 1,2)$ from the condition $\frac{\partial U_{eff}}{\partial \phi_{xi}} = 0$ for the effective potential of a connecting loop with the switching devices. Solving these coupled equations numerically, we obtain the total current, $I = n_c A' q \frac{k'}{m}$, as a function of f_x for a given $\tilde{f}$. When $f_x = 0$, the switching device is identical to a single Josephson junction with coupling energy E'_J of cross section A', i.e., the switch on-state, while the current vanishes at $f_x = 1/2$, i.e., the switch off-state regardless of the value of $\tilde{f}$.

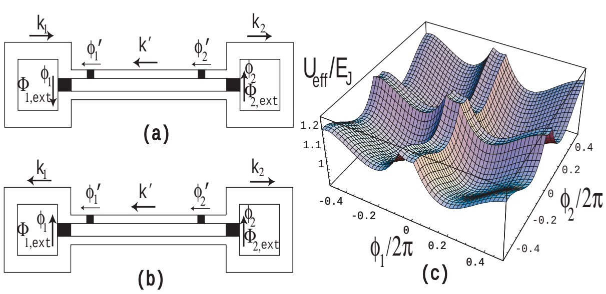

Fig. 2. (a) Coupled qubits in the same state (direction of current). Two Josephson junctions (small solid squares) denote switches. (b) Coupled qubits in different states. (c) Effective potential of the coupled qubits.

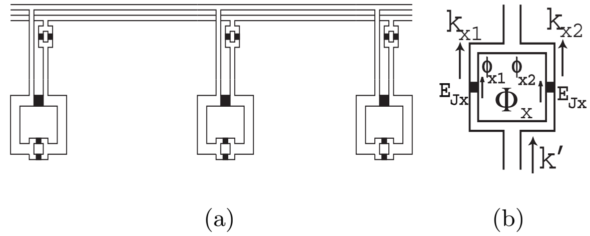

Fig. 3. (a) Part of the design for a scalable quantum computing device. (b) The detailed picture of a switching dc SQUID.

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