The inverse analysis of the enclosed cavity perturbation technique

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Abstract

Electrical conductivity can be conveniently measured by enclosed cavity perturbation technique in the microwave region. We studied the inverse analysis based on the double sphere model where the exact solutions of full Maxwell's equations can be calculated analytically. We found that the ranges of applicability of the skin depth regime and the depolarization regime were $k''a \gtrsim 10$ and $k''a \lesssim 1$, respectively.

Key words: microwave; measurement; Maxwell's equations; cavity perturbation

1. Introduction

In the so-called enclosed cavity perturbation technique for electrical conductivity measurement at microwave frequencies [1], one measures the change of the quality factor, Q, and that of the resonant frequency, f, by the introduction of a sufficiently small sample at the electric field antinode (E-meas.) or the magnetic field antinode (H-meas.) of a cavity resonator. The data analysis in the cavity perturbation technique is an inverse eigen value problem of Maxwell's equations and cannot be solved in general. However, under appropriate limiting conditions, some methods for the data analysis have been established and their reliabilities have been confirmed both experimentally and theoretically.
In the skin depth regime (SDR), where the skin depth of the sample, δ, is much smaller than the sample size, a, ($\delta \ll a$), the complex frequency shift is related to the surface impedance of the sample, ζ.

$$\frac{\Delta\hat{\omega}}{\omega_0} \approx \left[\frac{\Delta\hat{\omega}}{\omega_0}\right]_{\rm SDR} \equiv \frac{\Delta\bar{\omega}_0}{\omega_0} - iG\zeta, \tag{1}$$

where the so-called metallic shift, $\Delta\bar{\omega}_0/\omega_0$, and the geometrical factor, G, are usually determined by the dc conductivity, σ_{dc}, utilizing the Hagen-Rubens relation, $\zeta = \sqrt{(\mu/\epsilon)} \approx \sqrt{(\mu\omega_0/4\pi i\sigma_{dc})}$. On the other hand, in the depolarization regime (DPR), where the two characteristic lengths of the electromagnetic fields inside the sample, i.e. the skin depth, δ, and the wavelength, λ, are much larger than the sample size, a, ($\delta, \lambda \gg a$), Buranov and Shchegolev derived their approximating formula for an isotropic ellipsoidal sample in the E-meas., by regarding the electric field inside the sample as that obtained in the static problem [2].

$$\frac{\Delta\hat{\omega}}{\omega_0} \approx \left[\frac{\Delta\hat{\omega}}{\omega_0}\right]_{\rm DPR} \equiv -\frac{\gamma}{n}\,\frac{V_S}{V_C}\,\frac{\epsilon - 1}{\epsilon + 1/n - 1}. \tag{2}$$

Here, γ is the so-called resonator constant determined by the mode and the geometry of the resonator, V_S and V_C are the sample volume and the cavity volume, respectively, and n is the depolarization factor of the sample in the direction of applied electric field.
Although the ranges of applicability of these analysis methods (eqns. (1) and (2)) are obviously determined by the condition on the sample size stated above, the qualitative discussion as the experimental technique has not been discussed so far. In this paper, we studied the inverse analysis based on the double sphere model [3] where the exact solutions of full Maxwell's equations can be calculated analytically.

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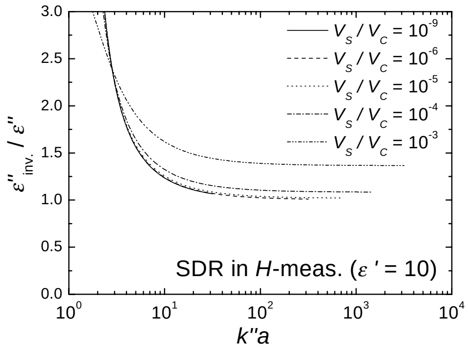

Fig. 1. The ratio of obtained $\epsilon''_{\text{inv.}}$ to original ϵ''. The data are pl

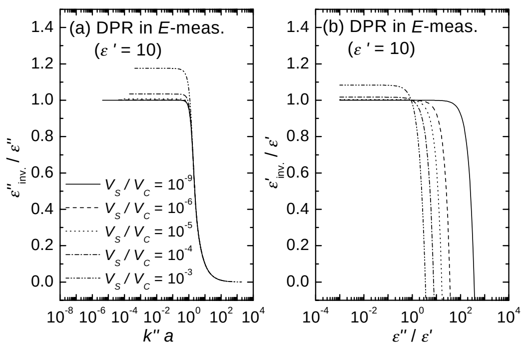

Fig. 2. The ratio of obtained $\epsilon''_{\text{inv.}}$ and $\epsilon'_{\text{inv.}}$ to original ϵ'' and ϵ'. The data are plotted as a function of $k''a \equiv 2\pi a/\delta$ and ϵ''/ϵ', respectively.

2. Result and Discussion

The geometrical configuration of the double sphere model is quite simple. A spherical sample with a uniform isotropic medium is inserted at the center of a spherical cavity resonator made of a perfect conductor. TM_{011} and TE_{011} mode of spherical cavity resonators correspond to the E-meas. and the H-meas., respectively, and the degeneracies of the modes are assumed to be resolved. The resonant frequency, f and the quality factor, Q, are determined as the functions of the sample radius, a, and the cavity radius, b, the complex permittivity, $\epsilon \equiv \epsilon' + i\epsilon''$, and the complex permeability, $\mu \equiv \mu' + i\mu''$, of the sample. The complex resonant frequency shift, $\Delta\hat{\omega}/\omega_0$ can be obtained by solving the complex eigen value problem of Maxwell's equations, which include ϵ, μ and the size ratio, $a/b \equiv (V_S/V_C)^{1/3}$, as the parameters. In the inverse analysis, we assume μ to be unity, calculate the permittivity, $\epsilon_{\text{inv.}} \equiv \epsilon'_{\text{inv.}} + i\epsilon''_{\text{inv.}}$, from $\Delta\hat{\omega}/\omega_0$ utilizing eqns. (1) and (2), and compare the results to the original permittivity, $\epsilon \equiv \epsilon' + i\epsilon''$.

The result of the inverse analysis in the SDR obtained from eqn. (1) are shown in Fig. 1. We plot the ratio, $\epsilon''_{\text{inv.}}/\epsilon''$ as a function of $k''a \equiv 2\pi a/\delta$. We estimate the geometrical factor G by Champlin Krongard model [4] ($G \approx (9/2)\gamma(V_S/V_C)/(k_0 a)$) and obtain $\epsilon''_{\text{inv.}}$ only from $\Delta(1/2Q)$ data assuming that $\epsilon' \ll \epsilon''$ for the normal metallic sample.

In the range of $k''a \gtrsim 10$, the ratio, $\epsilon''_{\text{inv.}}/\epsilon''$, approaches to a constant, ($\epsilon''_{\text{inv.}}/\epsilon'' \rightarrow$ *const.*), which is nearly unity. Therefore we can claim that the analysis by eqn. (1) for the SDR is valid for the range of $k''a \gtrsim 10$. The discrepancy of the constant from unity is due to the error of the geometrical factor, G, by Champlin Krongard model.

Figure 2 shows the results of the inverse analysis in the DPR obtained from eqn. (2). Again we plot the ratios, $\epsilon''_{\text{inv.}}/\epsilon''$ and $\epsilon'_{\text{inv.}}/\epsilon'$ as functions of $k''a$ and ϵ''/ϵ', respectively.

In the range of $k''a \gtrsim 1$, $\epsilon''_{\text{inv.}}/\epsilon''$ falls off dramatically from nearly unity to zero. This behavior is common for all filling factors, V_S/V_C, and we can claim that the analysis by eqn. (2) in the DPR is valid for the range of $k''a \lesssim 1$ as far as $\epsilon''_{\text{inv.}}$ is concerned. On the other hand, if we plot $\epsilon'_{\text{inv.}}/\epsilon'$ not as a function of $k''a$ but as that of ϵ''/ϵ', a cross point appears at $\epsilon''/\epsilon' \sim 1$. The ratio, $\epsilon'_{\text{inv.}}/\epsilon'$ falls off dramatically from nearly unity to negative value beyond $\epsilon''/\epsilon' \gtrsim 1$. This behavior is due to the change of the resonant frequency at the so-called depolarization peak, and we can claim that the analysis of $\epsilon'_{\text{inv.}}$ collapses in the range of $\epsilon''/\epsilon' \gtrsim 1$. That is, in general, the range of applicability of eqn. (2) is different between real and imaginary parts.

When $V_S/V_C \gtrsim 10^{-3}$, the perturbation analysis by eqn. (2) causes the discrepancies of the ratios, $\epsilon''_{\text{inv.}}/\epsilon''$ and $\epsilon'_{\text{inv.}}/\epsilon'$, from unity.

In conclusion, the inverse analysis based on the exact solutions of full Maxwell's equations are performed. We found that the range of applicability of the SDR and the DPR are $k''a \gtrsim 10$ and $k''a \lesssim 1$, respectively. For the estimation of the geometrical factors, it is necessary that $V_S/V_C \lesssim 10^{-3}$.

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