Self-Similar Conductance Fluctuations in Low Temperature Magneto-Transport of Coupled Dot Systems

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Abstract

Self-similar conductance fluctuations have been studied by means of low temperature magneto-resistance in order to clarify the fractal transport behavior in quantum coupled dot systems. We have observed a clear three-hold or four holds self-similar structure in the magneto-resistance of coupled dot systems. In our discussion based on saddle-point potential model, the self-similar and unstable periodic orbits can be expected by isochronous pitchfork bifurcations due to harmonic saddles in curved walls of the dot.

Key words: Self-Similar Conductance Fluctuations, Fractal Transport Behavior, Quantum Dots

Quantum chaos is expected as quasi-quantum behaviors in a classically non-integral system with a semi-classical limit, also with classical phase space structures blurred by the uncertainty principle. The statistical properties of quantum levels of chaotic systems were described in the stadium billiard[1]. Fractal behaviors are also observed in low temperature magneto-resistance in chaotic cavities[2,3]. These phenomena have been observed in open mesoscopic systems of small semiconductor circuits with length scale much shorter than both elastic and inelastic mean free paths but larger than the Fermi wave length. In this paper, we discuss on such fractal behaviors in a quantum dot system.

We measured a single quantum dot system and performed fractal analysis of the magneto-resistance measurement. The geometrical design of the split gate is shown in Fig. 1a, and the mobility and density of the two-dimensional electron gas (2DEG) are 2.5×10^5 cm^2 /Vs and 2.4×10^{11} cm^{-2}, respectively. The samples are cooled down with a Kelvinox ^{3}He/^{4}He dilution refrigerator, and performed a low power measurement using a lock-in amp technique. The magnetic field was applied perpendicular to the 2DEG of the dot.

The gate voltage has been applied to only plunger gate and this may cause a clear change both the shape of the dot and also that of the orbit of electron waves in the dot, as predicted previously [4]. If the voltages of the all gate are varied as the same manner, the effect for shrinking of the quantum point contacts is significantly larger than the change of the shapes in the dots. Then, the plunger gate is only controlling in this study without changing channel number in the dot. In the analysis for the fractal behaviors in the magneto-resistance, two methods have been used: self-similar scaling for hierarchical structure and box counting for volume dimension. The gate voltage dependence from -0.7 to -2.1 V is shown in Fig. 1b and several locations of magneto-resistance show a clear hierarchical structure. Therefore, we can estimate the fractal dimension

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by using self-similarity. Another fractal dimension can be also considered from the volume in various scales (gbox countingh) [4].

The results for fractal analysis on both methods are shown in Figs. 2 and 3. The fractal property is affected by not only changing the gate voltage, but also the magnetic field, as shown in Fig. 3. Comparing both results, the dimension determination from box counting is not equal to that from the self-similar method. One of reason for this discrepancy can be considered that self-similar scaling seems to be able to neglect the non-fractal signals in the analysis. As the transport characteristics would vary with both magnetic field and gate voltage at the plunger gate, the fractal dimension including non-fractal signal also vary frequently with these two parameters. On the other hand, since the specific pattern in the hierarchical structure is important for the self-similar method, we can obtain a reasonable fractal dimension by use of self-similar scaling. In addition, this kind hierarchical structure can be discussed in scattering process at the saddle-point potential as well as a kinetic process in the mixed phase space in chaotic cavities. The self-similar and unstable periodic orbits can be expected by isochronous pitchfork bifurcations due to harmonic saddles in curved walls of the dot [5]. Based on this model, we can calculate a hierarchical magneto-resistance pattern corresponding suitable experimental conditions. Probably, a monotonic change of the fractal dimension must be applicable for such saddle potential scatterings.

In conclusion, we have made the fractal dimension analysis on the data from magneto-resistance measurements. The single dot shows a complex fractal behavior in the transport phenomena when we change the parameters of the gate voltage and the magnetic field. Although we have no clear correlation between box-counting and self-similar scaling for the determination of the fractal dimension, self-similar scaling is rather resonable for the analysis of the fractal behavior.

References

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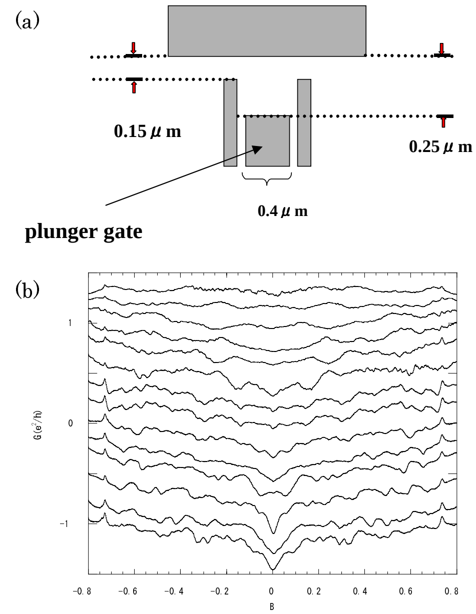

Fig. 1. (a) The gate pattern of the single dot system. (b) The gate voltage dependence of the low temperature magneto-resistance. The gate voltage is increased from -0.7 to -2.1 V.

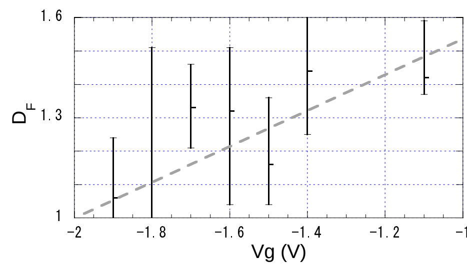

Fig. 2. Estimated fractal dimension for self-similar scaling.

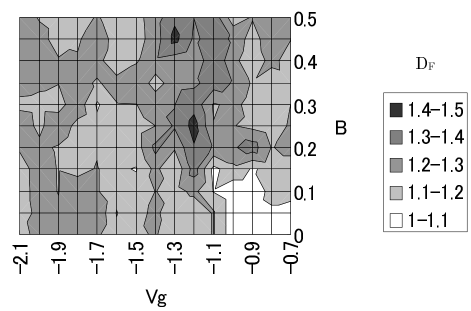

Fig. 3. Contour mapping of fractal dimension for box counting method.