

# The properties of Fulde-Ferrell-Larkin-Ovchinnikov state in $d$ -wave superconductors

K. Maki <sup>a,b</sup> H. Won <sup>c,1</sup>

<sup>a</sup> *Max-Planck-Institute for the Physics of Complex Systems, Nöthnitzer Str.38, 01187 Dresden, Germany*

<sup>b</sup> *Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-0484, USA*

<sup>c</sup> *Department of Physics, Hallym University, Chunchon, 200-702, South Korea*

---

## Abstract

There is renewed interest in the Fulde-Ferrell-Larkin-Ovchinnikov(FFLO) state in  $d$ -wave superconductors in layered compounds. In strong contrast 3D  $s$ -wave superconductors where a stripe-like phase is favored, a square lattice like structure is found in quasi 2D  $d$ -wave superconductors when  $t(= T/T_c) \leq 0.06$ . At higher temperatures a stripe-like state appears to be more favored. Indeed there have been reported that FFLO state is seen in  $\beta$ -(ET)<sub>2</sub> salts and  $\lambda$ -(BEDT)<sub>2</sub>GaCl<sub>4</sub>. We shall review briefly our recent works on FFLO state in  $d$ -wave superconductors. In particular we find simple expressions for the density of states, NMR relaxation rate, the specific heat and the thermal conductivity which should be accessible experimentally.

*Key words:*  $d$ -wave superconductors, Pauli paramagnetism Periodic state

---

In a spin singlet superconductors a magnetic field couples both spin and orbital motion of electrons. When the coupling to spin dominates(i.e. the Pauli limiting) the superconductivity enters into a new state with a periodic modulation of  $\Delta(\mathbf{r})$ [1,2]. This new state is called Fulde-Ferrell-Larkin-Ovchinnikov(FFLO) state. However, in conventional  $s$ -wave superconductors the realization of FFLO state is thought almost impossible. First of all the quasiparticle mean free path  $l$  should be much longer than the coherence length  $\xi$ . Otherwise the impurity scattering destroys FFLO easily. Second in order to have the dominance of the Pauli paramagnetism the Ginzburg Landau parameter  $\kappa$  has to be much larger than one. These two conditions are difficult to be met in conventional  $s$ -wave superconductors.

The appearance of the unconventional superconductors open up a new window. First of all many of these systems are extremely clean. Further  $\kappa$  ranges in general  $10 \sim 100$ . In some of organic molecular superconductors in a planar magnetic field the situation looks very favorable [3].

Further in  $d$ -wave superconductors the field region where FFLO state exists is much more extended [3,4]. Also FFLO has two different structures for  $t(= T/T_c) < 0.06$  and  $t > 0.06$ . For  $t < 0.06$   $\Delta(\mathbf{r}) \sim \cos(qx) + \cos(qy)$  while for  $t > 0.06$  in the most region  $\Delta(\mathbf{r}) \sim \cos \frac{q}{\sqrt{2}}(x+y)$  or  $\Delta(\mathbf{r}) \sim \cos \frac{q}{\sqrt{2}}(x-y)$  [4]. Here  $q$  is the momentum associated with the order parameter  $\Delta(\mathbf{r})$ . In Fig.1 we show the temperature dependence of  $q$  in the reduced unit  $v|q|/2h \equiv p$  versus  $t$  and  $h = \mu_B H$  and  $v$  is the Fermi velocity in the conducting plane. You note  $p = 1$  at  $t = 0$ . Then  $p$  increases with increasing  $t$  and has a maximum around  $t \sim 0.2$ . At the first order phase transition point,  $t = 0.06$ ,  $p$  jump into a new value  $p \sim 1.22$ .

In the following we discuss some of the remarkable features.

---

<sup>1</sup> Corresponding author. Present address: Department of Physics, Hallym University, Chunchon, 200-702, South Korea  
E-mail: hkwon@hallym.ac.kr

HW acknowledges the support from the Korean Science and Engineering Foundation(KOSEF) through the Grant No. 1999-2-114-005-5.

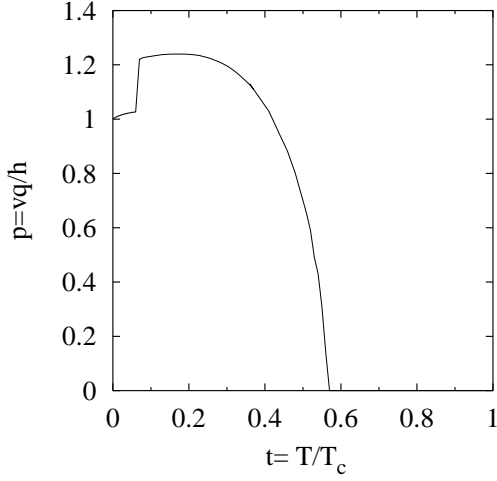


Fig. 1. The temperature dependence of  $q$  in the reduced unit  $p \equiv v|q|/2h$  versus  $t = T/T_c$  is shown.

(a)  $t < 0.06$ , we call this region FFLO I. Here  $p$  cannot be very large in FFLO I though  $p > 1$  except  $t = 0$ . This region is characterized by a square lattice like  $\Delta(\mathbf{r})$  [5]. The quasiparticle density of states (DOS) is given by [5]

$$\begin{aligned} \frac{N(E)}{N_0} (\equiv g(E)) &\simeq 1 + \frac{\Delta^2}{4} \sum_{\pm} \left\langle \frac{\cos^2(2\phi)}{[E \pm h + \frac{vq}{2} \cos \phi]^2} \right\rangle \\ &= 1 + \frac{\Delta^2}{2h^2} J\left(\frac{E}{h}, p\right) \end{aligned} \quad (1)$$

and

$$J(\varepsilon, p) = \frac{1}{2} \sum_{\pm} \text{Re} \left\{ \frac{|\varepsilon \pm 1|}{[(\varepsilon \pm 1)^2 - p^2]^{3/2}} + \frac{4}{p^4} \left[ (3(\varepsilon \pm 1)^2 - \frac{p^2}{2}) - (3(\varepsilon \pm 1)^2 - 2p^2) \frac{|\varepsilon \pm 1|}{\sqrt{(\varepsilon \pm 1)^2 - p^2}} \right] \right\} \quad (2)$$

and

$$\begin{aligned} J(0, p) &= \text{Re} \frac{1}{(1 - p^2)^{3/2}} + \frac{4}{p^4} \left( 3 - \frac{p^2}{2} - (3 - 2p^2) \times \right. \\ &\quad \left. \text{Re} \frac{1}{\sqrt{1 - p^2}} \right) = \frac{4}{p^4} \left( 3 - \frac{1}{2} p^2 \right) \end{aligned} \quad (3)$$

At low temperatures, many quantities can be expressed in terms of

$$g(0) = 1 + 2 \left( \frac{\Delta}{h} \right)^2 \frac{1}{p^4} \left( 3 - \frac{1}{2} p^2 \right) \quad (4)$$

$$\frac{T_1^{-1}}{T_{1n}^{-1}} = (g(0))^2, \quad \frac{\chi}{\chi_n} = g(0) \quad (5)$$

$$\frac{C_s}{\gamma T} = g(0), \quad \frac{\kappa_{xx}}{\kappa_n} = (g(0))^{-1}$$

where  $T$ ,  $\chi$ ,  $C_s$ , and  $\kappa_{xx}$  are the nuclear spin lattice relaxation rate, spin susceptibility, the specific heat and the planar thermal conductivity. Except for the thermal conductivity, all other quantities increase in FFLO state. More generally

$$\frac{\kappa_{xx}}{\kappa_n} = \frac{3}{2\pi^2} \int_0^\infty dx x^2 g^{-1}(Tx) \text{sech}^2\left(\frac{x}{2}\right) \quad (6)$$

(b)  $0.06 < t < 0.56$  we call this region FFLO II. According to Shimahara there is a narrow region, where a square lattice may be realized. But we ignore this small region for simplicity. Then we have

$$\frac{N(E)}{N_0} (\equiv g(E)) = 1 + \frac{\Delta^2}{2h^2} K\left(\frac{E}{h}, p\right) \quad (7)$$

and

$$\begin{aligned} K(\varepsilon, p) &= \frac{1}{2} \sum_{\pm} \left\{ \frac{4}{p^4} \left( \frac{p^2}{2} - 3(\varepsilon \pm 1)^2 \right) \right. \\ &\quad \left. + \frac{4}{p^4} (3(\varepsilon \pm 1)^2 - 2p^2) \text{Re} \frac{|\varepsilon \pm 1|}{\sqrt{(\varepsilon \pm 1)^2 - p^2}} \right\} \end{aligned} \quad (8)$$

As in the region I, there appear two peaks. Also we find

$$K(0, p) = \frac{4}{p^4} \left[ \left( \frac{p^2}{2} - 3 \right) + (3 - 2p^2) \text{Re} \frac{1}{\sqrt{1 - p^2}} \right] \quad (9)$$

This gives

$$g(0) = 1 + 2 \frac{\Delta^2}{h^2} \frac{1}{p^4} \left[ \frac{p^2}{2} - 3 + (3 - 2p^2) \text{Re} \frac{1}{\sqrt{1 - p^2}} \right] \quad (10)$$

From this we get  $T_1^{-1}/T_{1n}^{-1}$  etc., though perhaps the approximation  $T \ll h, vq/2$  will not be as good as in regime I.

Recently possible FFLO state in  $\kappa$ -(ET)<sub>2</sub>Cu(NCS)<sub>2</sub> [6] and  $\lambda$ -(BEDT)<sub>2</sub>GaCl<sub>4</sub> [7] has been reported. Indeed in [7] it is shown that the thermal conductivity suddenly decreases in entering FFLO state, which is consistent with the present result.

## References

- [1] P. Fulde and R. A. Ferrell, Phys. Rev. 185 (1964) A550.
- [2] A.I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP 20 (1965) 762.
- [3] K. Maki and H. Won, Czech. J. Phys. 46, S2 (1996) 1033.
- [4] H. Shimahara, J. Phys. Soc. Jpn. 67, 1872 (1997).
- [5] H. Won and K. Maki, Physica B (in press).
- [6] M.S. Nam et al., J. Phys. Cond. Mat. 11 (1999) L477; J. Singleton et al., cond-mat/0105335.
- [7] M. A. Tanatar et al., cond-mat/0205239, Phys. Rev. B(submitted).