Continuous Vortices and Collective Excitations in Ferromagnetic Spinor Bose-Einstein Condensates

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Abstract

Bose-Einstein condensates (BEC) realized in alkali atomic gases with the hyperfine state $F = 1$ keep "spin" states degenerate and active under an optical trap. These systems, so-called spinor BEC are analogous to the A-phase of the superfluid ^{3}He. Among various topological structures, the Mermin-Ho and Anderson-Toulouse vortices are proposed in superfluid ^{3}He-A phase. We demonstrate by solving the extended Gross-Pitaevskii equation that these topological structures are thermodynamically stable in ferromagnetic spinor BEC under rotation. Furthermore, we show the collective modes for such the vortices within Bogoliubov theory.

Key words: Bose-Einstein condensates; Vortex; Texture; Bogoliubov theory;

1. Introduction

The atomic gases with the hyperfine spin $F = 1$ have been Bose-condensed via the optical methods as named a spinor Bose-Einstein condensate (BEC)[1,2], which provides a research field to investigate the exotic topological structuers. The topological defects, such as skyrmion, Mermin-Ho texture, and monopole, play an essential role in various fields of the physics, ranging from condensed matter physics to high energy physics. The theoretical study of a spinor BEC was intiated by Ohmi and Machida[3], and Ho[4], who pointed out the richness of the vortices and the topological defects.

In our previous works[5], for the ferromagnetic case, the thermodynamic stability of the topological defect, Mermin-Ho(MH) vortex, has been shown by considering the effect of the external rotation. In addition to this continuous vortex, for the ferromagnetic case, the $\langle 1, 0, -1\rangle$ vortex, which the polar state is localized in the vortex core, also is favored. In the present paper, we investigate the stability of the MH vortex, namely, the anormalous modes which present the instability of this state.

Since we consider the Bose condensed system in the ferromagnetic $F=1$ situation, the order parameters are characterized by the hyperfine sublevels $m_F = 1, 0, -1$. The Mermin-Ho (MH) vortex then is described by

$$\begin{pmatrix} \phi_1(\mathbf{r}) \\ \phi_0(\mathbf{r}) \\ \phi_{-1}(\mathbf{r}) \end{pmatrix} = \sqrt{n(r)} \begin{pmatrix} \cos^2 \dfrac{\beta}{2} \\ \sqrt{2} e^{i\varphi} \sin \dfrac{\beta}{2} \cos \dfrac{\beta}{2} \\ e^{2i\varphi} \sin^2 \dfrac{\beta}{2} \end{pmatrix} \quad (1)$$

where the bending angle $\beta(r)$ is $0 \leq \beta(r) \leq \pi$ and φ signifies the polar angle in polar coordinates. $n(r)$ is the total density profile, which is given by solving the Gross-Pitaevskii equation[5]. The spin direction is denoted by the l-vector and is given as $\hat{l}(r) = \hat{z}\cos\beta + \sin\beta(\cos\phi\hat{x} + \sin\phi\hat{y})$ where β varies from $\beta(0) = 0$ to $\beta(R) = \frac{\pi}{2}$ $(= \pi)$ for MH (Anderson-Toulouse (AT)) (R is the outer boundary of the cloud). Thus the spin moment is flared out to the radial direction and at the circumference it points outward for MH and downwards for AT.

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2. Method and Results

The time-dependent Gross-Pitaevskii (GP) equation in a spinor BEC is obtained for the condensate wave functions ψ_j $(j=1,0,-1)$[3,4] as

$$i\hbar\frac{\partial}{\partial t}\psi_i(\mathbf{r},t) = \left[\left\{h_i(\mathbf{r}) + g_n\sum_k|\psi_k|^2\right\}\delta_{ij} + \frac{g_s}{2}\sum_\alpha\langle\hat{F}_\alpha\rangle(\hat{F}_\alpha)_{ij}\right]\psi_j(\mathbf{r},t), \qquad (2)$$

where $\langle\hat{F}_\alpha\rangle=\sum_{l,m}\psi_l^*(\hat{F}_\alpha)_{lm}\psi_m$ and $h_i(\mathbf{r})=-\frac{\hbar^2\nabla^2}{2m}-\mu_i+V(\mathbf{r})-\mathbf{\Omega}\cdot(\mathbf{r}\times\mathbf{p})$ is one-body Hamiltonian, the "density" interaction channel $g_{\rm n}=\frac{4\pi\hbar^2}{m}\cdot\frac{a_0+2a_2}{3}$, the "spin" interaction channel $g_{\rm s}=\frac{4\pi\hbar^2}{m}\cdot\frac{a_2-a_0}{3}$. The scalar field $V(\mathbf{r})=\frac{1}{2}m\omega_r^2r^2$ is the external confinement potential such as an optical potential. Here we assume uniformity along z-axis and take the external rotation as $\mathbf{\Omega}=\Omega\cdot\hat{z}$. The following results are calculated with the spin interaction $g_s=-0.02g_n$ and $\Omega=0.3\omega_r$.

We derive the time-independent GP equation from Eq.(2), corresponding to the equation of motion for $\phi_j(\mathbf{r})$ defined as the stationary part of the condensate wave functions $\psi_j(\mathbf{r},t)$. The density profiles of the continuous vortex $|\phi_j(\mathbf{r})|^2$ at $M/N=0,0.5,1.0$ are shown in Fig.1(a), where the total magnetization is given as $M/N=\int d\mathbf{r}\sum_j j|\phi_j|$. At $M/N=1$, the density profile is completely equivalent to the vortex-free state in the scalar BEC and the spin moments are polarized. As the magnetization decreases, the other components with the winding number grows up in the circumference of the spin-1 condensate while the local magnetization in the condensate surface changes continuously from positive to negative values: the spin direction for $M/N\sim0.5$ orients horizontally (MH texture) and for $M/N\sim0$ points down (AT texture). Thus the spin texture can be controlled by merely changing the total magnetization. Furthermore, we note that these topological structures are never stable under no rotaion.

In order to verify the "local" stability, we consider the equation of motion for the small perturbation: $\psi_j(\mathbf{r},t)=\phi_j(\mathbf{r})+u(\mathbf{r},j)e^{-i\varepsilon_q t/\hbar}+v^*(\mathbf{r},j)e^{i\varepsilon_q t/\hbar}$. By retaining terms up to first order in $u(\mathbf{r})$, $v(\mathbf{r})$, we derive the Bogoliubov equation, i.e. the equation of motion for the collective excitations[5]. The appearance of negative ε_q implies the local intrinsic instability of the relevant vortex in the energy landscape.

M-dependence of the negative eigenvalues is shown in Fig.1(b). There are two anormalous modes with $q_\theta=-1,-2$ under the low rotaion drive where q_θ corresponds to the quantum number of the θ-direction. In $M/N=1$ case, where the spin moment of the condensate is polarized, both modes corresponds to the spin wave excitation: the mode with $q_\theta=-1$ is the transverse spin wave (TSW) mode and the mode with $q_\theta=-2$ is the longitudinal spin wave (LSW) mode. Here, as the spin interaction g_s/g_n increases, the LSW mode confines to the condensate surface. At $M/N\sim0.5$ situation with the MH texture, these two modes become the TSW (LSW) mode localized near the center of the trap. In $M/N<0.5$ region, the negative eigenvalue appears in $q_\theta=-1$, which discribes the instability of the continuous vortex. As the external rotation Ω increases, the eigenvalue of this anormalous mode shifts to the positive value and in the wide area of M/N the continuous vortex has the local stability.

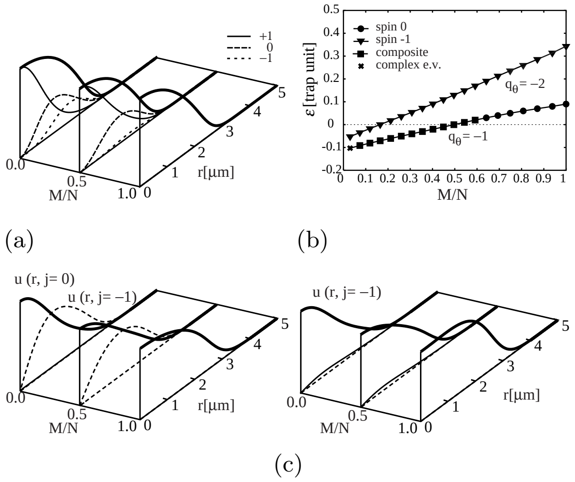

Fig. 1. (a) The density profiles of the condensate at $\Omega=0.3$ and $M=1.0$, 0.5, 0.0. The bold line show the total density $\sum|\phi_j|^2$. (b) The two anormalous modes are plotted as the function of the total magnetizatoin M/N. (c) The wave function of the mode with $q_\theta=-1$ (left) and $q_\theta=-2$ (right). The bold line corresponds to the wave function of the spin-0 component (left), $u(r,j=0)$, and the spin-−1 component (right), $u(r,j=-1)$.

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