A Theoretical Study on Orbital Magnetism of Mesoscopic Ring Systems

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Abstract

Spatial distribution of current and resultant magnetization of mesoscopic ring systems with finite width are investigated in various regimes of temperature and magnetic field based on the exactly solvable model. The Aharonov-Bohm (AB) oscillation is observed in low temperatures when electrons occupy only one Landau sublevel. When several Landau sublevels are occupied, the coherence of the AB oscillation is destroyed and mesoscopic fluctuations appear instead. The finite Landau sublevels leads to also de Haas-van Alphen oscillations in relatively low temperatures. In high temperatures, the Landau diamagnetism is observed like in the disk systems.

Key words: orbital magnetism; Aharonov-Bohm oscillation; de Haas-van Alphen oscillation; mesoscopic ring with finite width

Interference effects in mesoscopic ring systems under magnetic fields lead to the Aharonov-Bohm (AB) oscillations in physical quantities, such as conductance, persistent current and consequently orbital magnetism. The AB oscillations are observed in a mesoscopic ring only when its radius is much larger than its width [1,2]. Effects of the finite width of rings are important for understanding the AB oscillations in actual ring systems. Tan and Inkson presented a two-dimensional ring model, which can be solved exactly and calculated conductance, magnetization and persistent current at $T = 0$ [3].

In this paper, we investigate the magnetization and spatial distribution of current based on the same exactly solvable model, looking on the wide range of temperatures and magnetic fields. The spatial dependence of current clearly reflects the coherence of electrons in the real space and shows characteristic features in each regions [4]. We focus on narrow width ring and study the effects of width on the orbital magnetism.

We consider the exactly solvable two-dimensional ring model [3]. Electrons are confined in xy-plane and uniform magnetic fields are applied in z-axis. The Hamiltonian is given by,

$$H = \frac{1}{2m}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 + \frac{\hbar^2}{2m}\frac{A_0}{r^2} + \frac{m}{2}\omega_0^2 r^2. \tag{1}$$

Here we use the two-dimensional polar coordinate. By using symmetric gauge, the eigen function and eigenvalue are obtained as,

$$\Psi_{n,\alpha}(\mathbf{r}) = \frac{1}{\sqrt{2\pi}}\, \mathrm{e}^{\mathrm{i}\alpha\theta}\, R_{n,\alpha}(r) \tag{2}$$

$$R_{n,\alpha}(r) = \frac{1}{l}\sqrt{\frac{\Gamma(n+1)}{\Gamma(n+\gamma+1)}}\, \mathrm{e}^{-\tilde{r}^2/2}\, \tilde{r}^{\gamma}\, \mathrm{L}_n^{(\gamma)}(\tilde{r}^2), \tag{3}$$

$$E_{n,\alpha} = \frac{1}{2}\hbar\omega_c\alpha + \frac{1}{2}\hbar\omega(2n+1+\gamma), \tag{4}$$

where $l = \sqrt{\frac{\hbar}{m\omega}}, \tilde{r} = r/\sqrt{2}l, \omega = \sqrt{\omega_c^2 + 4\omega_0^2},\ \gamma = \sqrt{\alpha^2 + A_0}$, and $\omega_c = \frac{eH}{mc}$. The quantum number, $\alpha = 0, \pm1, \pm2, \cdots$, denotes the angular momentum and $n = 0, 1, 2, \cdots$ does the number of nodes of the wave function in the radial direction. We take $e > 0$.

Magnetization in grand canonical ensemble is calculated by,

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$$M = -\sum_{n,\alpha} \frac{\partial E_{n,\alpha}}{\partial H} f(E_{n,\alpha}). \quad (5)$$

The spatial dependence of the field induced persistent current in the θ-direction, $J_\theta(r)$, is given by

$$J_\theta(r) = -j_0 \sqrt{\frac{2\omega}{\omega_0}} \sum_{n,\alpha} l^2 R_{n,\alpha}(r)^2 f(E_{n,\alpha}) \left(\frac{\alpha}{\tilde{r}} + \frac{\omega_c \tilde{r}}{\omega}\right), \quad (6)$$

where $j_0 \equiv e\omega_0 \xi / 4\pi$, and $\xi \equiv \sqrt{\hbar / m\omega_0}$.

This current and the magnetization satisfy the equation,

$$M = \frac{1}{2c} \int \mathrm{d}\mathbf{r}\; r J_\theta(r) = \frac{\pi}{c} \int \mathrm{d}r\; r^2 J_\theta(r). \quad (7)$$

For example, we show the results of the magnetic field dependence of the magnetization of a narrow ring with $A_0 = 250000$ and 500 electrons in Fig. 1; Fig. 1 (a) shows the results for wide range of magnetic fields and several choices of temperatures, Fig. 1 (b) for $1.00 \le \omega_c/\omega_0 \le 1.12$ at $T = 0$ and Fig. 1 (c) for $5.90 \le \omega_c/\omega_0 \le 6.02$ at $T = 0$. In this system electrons are in $n = 0, 1, 2$ and 3 states at $T = 0$ and $\omega_c = 0$. Effective edges of the ring determined by the chemical potential are $R_{\mathrm{in}}/\xi = 20.4$ and $R_{\mathrm{out}}/\xi = 24.5$, respectively. Fig. 1 (a) indicates a slow and a rapid oscillations and the Landau diamagnetism. The slow oscillation is seen to be de Haas-van Alphen (dHvA) oscillation caused by the change of n_{max}, which remains for relatively high temperatures. The rapid one exists only in the low temperatures, which is shown in Fig. 1 (b) and (c) in detail. The regular AB oscillations are observed clearly only for $\omega_c/\omega_0 > 5.96$; in this region only $n = 0$ states are occupied. For $\omega_c/\omega_0 < 5.96$, there exist electron transfers between different n states by change of magnetic fields. These transfers cause the decoherence of the AB oscillations and lead to mesoscopic fluctuations instead, which is seen in disk systems [4].

The spatial distributions of the persistent current of the system for $\omega_c/\omega_0 = 0.1$ and several choices of temperatures are shown in Fig. 2. In the low temperatures the current is induced largely in the ring. The current strongly depends on magnetic fields and becomes plus and minus. For $0.1 < k_B T/\hbar\omega_0 < 1$, the current has regular spatial dependence, reflecting nodes of wave functions near the fermi energy. For $k_B T/\hbar\omega_0 = 2$, the current exists only around edges, which leads to the Landau diamagnetism like in the disk cases [4].

In conclusion, the orbital magnetism in the narrow ring system shows the AB oscillations or fluctuations in low temperatures and diamagnetism caused by edge currents in high temperatures. The dHvA oscillations are also observed by the existence of finite Landau sublevels. The condition of the regular AB oscillation is that only one Landau sublevel is occupied.

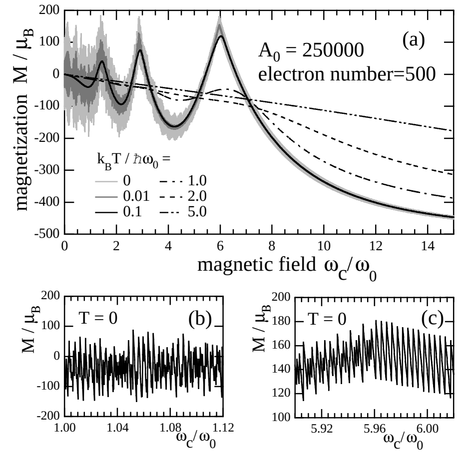

Fig. 1. The magnetic field dependence of the magnetization of the ring with $A_0 = 250000$ and electron number $= 500$ for (a) $\omega_c/\omega_0 \le 15$ and several choices of temperatures, (b) $1.00 \le \omega_c/\omega_0 \le 1.12$ at $T = 0$ and (c) $5.90 \le \omega_c/\omega_0 \le 6.02$ at $T = 0$.

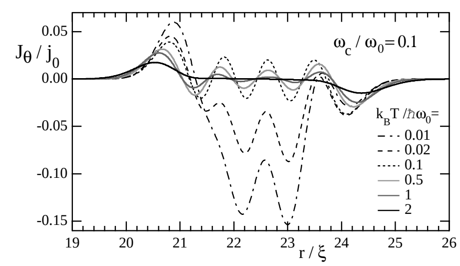

Fig. 2. The spatial distribution of the persistent current of the ring with $A_0 = 250000$, electron number $= 500$ and $\omega_c/\omega_0 = 0.1$.

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