Gauge theory of composite fermions

Ikuo Ichinose [a], Tetsuo Matsui [b,1]

[a] *Department of Electrical and Computer Engineering, Nagoya Institute of Technology, Nagoya, 466-8555 Japan*

[b] *Department of Physics, Kinki University, Higashi-Osaka, 577-8502 Japan*

Abstract

We explain the success of Jain's composite fermion theory for quantum Hall systems at the filling factor $\nu = p/(2pq \pm 1)$ by applying the gauge theory of particle-flux separation. At temperatures $T < T_{\rm PFS}(\nu)(\simeq 5.5 \sim 6.2{\rm K}$ for $\nu = 1/2$), the charge and Chern-Simons flux degrees of freedom of electrons separate. We call quasiparticles carrying each quantum number chargeons and fluxons, respectively. Bose condensation of fluxons below $T_{\rm BC}(< T_{\rm PFS})$ justifies the (partial) cancellation of the external magnetic field, as assumed in the composite-fermion theory. The resistivity ρ_{xx} starts to behave differently below $T_{\rm PFS}$ due to the dynamical gauge field.

Key words: quantum Hall effect; composite fermions; lattice gauge theory; confinement-deconfinement transition

The picture of composite fermions (CFs) by Jain[1] explains the essential properties of the fractional quantum Hall effects (QHE) at the filling factors $\nu = p/(2pq + 1)$ (p, q; positive integers). In the approach by the Chern-Simons (CS) gauge theory, the validity of CF picture relies upon the possibility that the CS fluxes attached to each CF cancel out the external magnetic field partly as assumed in the mean-field theory (MFT). It is important to investigate this possibility based on the microscopic theory.

In Ref.[2], we considered $\nu = 1/2$ and showed that this cancellation really takes place for temperatures T *below* a certain critical temperature $T_{\rm PFS}$ via the mechanism of particle-flux separation (PFS). There, we introduced the chargeon and fluxon operators to describe the charge and flux degrees of freedom of an electron, respectively. They are confined into electrons by a U(1) gauge field at $T > T_{\rm PFS}$, and PFS is characterized as a deconfinement phenomenon of the gauge dynamics of this gauge field. In [3], we generalized the PFS theory to $\nu = p/(2pq + 1)$. The PFS is a counterpart of the charge-spin separation (CSS) [4] in high-T_C cuprates. In this paper, we explain PFS concisely, and discuss the temperature dependence of the resistivity ρ_{xx}.

[1] Corresponding author. E-mail:matsui@phys.kindai.ac.jp

To study the nonperturbative effect, we put the electron system in an external magnetic field $B^{\rm ex}$ onto a 2D lattice with the lattice spacing $a \sim \ell \equiv (eB^{\rm ex})^{-1/2}$. (Hereafter we use the unit $a = 1$.) We write the electron operator C_x at the site x as

$$C_x = \exp\left[2iq\sum_y \theta_{xy}\phi^\dagger_y\phi_y\right]\phi_x\eta_x, \tag{1}$$

where θ_{xy} is the multivalued angle function on a lattice, ϕ_x is the bosonic fluxon operator, η_x is the fermionic chargeon operator. The phase factor describes that each electron carries $2q$ units of CS flux quanta. (See Fig.1)

The filling factor is given by $\nu = 2\pi n/(eB^{\rm ex})$ where $n \equiv \langle C^\dagger_x C_x\rangle$. To maintain the fermionic anticommutation relations of C_x, one needs to impose the following local constraint:

$$\eta^\dagger_x\eta_x = \phi^\dagger_x\phi_x. \tag{2}$$

The Hamiltonian is written in terms of η_x and ϕ_x as

$$H_{\eta\phi} = -\frac{1}{2m}\sum_x\sum_{j=1}^{2}\Big(\eta^\dagger_{x+j}\phi^\dagger_{x+j}W_{x+j}M_{x+j} \times M^\dagger_x W^\dagger_x\phi_x\eta_x + h.c.\Big) + H_{\rm int}(\eta^\dagger_x\phi^\dagger_x\phi_x\eta_x),$$

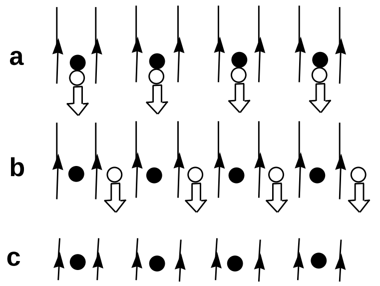

Fig. 1. Illustration of PFS. (a) Electrons in magnetic field $B^{\rm ex}$. Thin arrows are $B^{\rm ex}$, black circles are chargeons η_x, white circles are fluxons ϕ_x, and thick white arrows are CS fluxes. See eq.(1). (b) In PFS states, chargeons and fluxons dissociate. (c) In FQHE states, fluxons form Bose condensate and the resulting uniform CS field cancels $B^{\rm ex}$ partly. Chargeons feel the residual field ΔB (thin arrows).

$$W_x = \exp\Big[2iq\sum_y \theta_{xy}(\phi_y^\dagger\phi_y - n)\Big],$$
$$M_x = \exp\Big[i\sum_y \theta_{xy} n\big(2q - \frac{1}{\nu}\big)\Big]. \qquad (3)$$

$H_{\rm int} = g\sum_{xj} C^\dagger_{x+j}C_{x+j}C^\dagger_x C_x$ $(g > 0)$ is the short-range part of Coulomb repulsion. C_x and $H_{\eta\phi}$ are invariant under the U(1) "local gauge transformation",

$$(\eta_x, \phi_x) \to (e^{i\alpha_x}\eta_x, e^{-i\alpha_x}\phi_x) \text{ for each } x. \qquad (4)$$

We employ the path-integral formalism and respect the constraint (2) by introducing the Lagrange multiplier field λ_x. We introduce a complex auxiliary field V_{xj} on the link $(x, x+j)$ and decouple the term in $H_{\eta\phi}$;

$$\eta^\dagger_{x+j}\phi^\dagger_{x+j}W_{x+j}M_{x+j}M^\dagger_x W^\dagger_x\phi_x\eta_x$$
$$\to V_{xj}J_{xj} + H.c. - |V_{xj}|^2$$
$$J_{xj} \equiv \gamma\phi^\dagger_x W_x W^\dagger_{x+j}\phi_{x+j} + \gamma^{-1}\eta^\dagger_{x+j}M_{x+j}M^\dagger_x\eta_x. \qquad (5)$$

Under (4), $V_{xj} \to e^{-i\alpha_x}V_{xj}e^{i\alpha_{x+j}}$, hence the phase A_{xj} of $V_{xj} \simeq V_0\exp(iA_{xj})$ is regarded as the U(1) gauge field. A_{xj} mediates interaction among chargeons and fluxons. At low energies, $A_{x\mu} \equiv (\lambda_x, A_{xj})$ become dynamical as a result of "renormalization" (radiative corrections) by high-energy modes. At low energies, there are two possible realizations of this U(1) gauge dynamics: (i) a deconfinement phase where the fluctuations of $A_{x\mu}$ are weak, and chargeons and fluxons are deconfined and behave as quasi-free particles, or (ii) a confinement phase where the fluctuations are strong and chargeons and fluxons are confined into the original electrons. The PFS is nothing but the deconfinement phenomenon (i).

Following the method developed in CSS [4] and lattice gauge theory, we find that there is a confinement-deconfinement phase transition at $T_{\rm PFS}(\nu)$, below which the deconfinement phase, hence PFS, is realized.

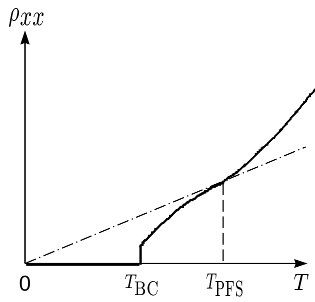

Fig. 2. Illustration of ρ_{xx}. Below $T_{\rm PFS}$, it exhibits a new behavior. Below $T_{\rm BC}$, $\rho_{xx} = 0$.

Numerical estimation of $T_{\rm PFS}$ for $\nu = 1/2$ [3] gives

$$T_{\rm PFS} = 5.5 \sim 6.2{\rm K} \quad \text{for} \quad g = (0.1 \sim 1) \times \frac{e^2}{\epsilon\ell}, \qquad (6)$$

where $a = \ell$, $B^{\rm ex} = 10[{\rm T}], m = 0.067\, m_{\rm electron}, \epsilon = 13$. Then $\gamma = 0.96 \sim 0.69$ and the masses of chargeon and fluxon at $T = 0$ are $m_\eta \equiv \gamma V_0^{-1}m = (6.6 \sim 4.8)m, m_\phi \equiv \gamma^{-1}V_0^{-1}m = (7.2 \sim 10.1)m$.

One expects that bosonic fluxons may Bose condense below $T_{\rm BC}(< T_{\rm PFS})$. At $\nu = p/(2pq \pm 1)$, the *uniform* CS field generated by the condensation of fluxons partly cancels the uniform $B^{\rm ex}$. Chargeons feel the residual field $\Delta B = B^{\rm ex} - B_\phi = \pm 2\pi n/(ep)$, and fill the p Landau levels of ΔB, giving rise to IQHE. This observation obviously implies that the chargeons are nothing but Jain's CFs[1].

Concerning to the physical properties below $T_{\rm PFS}$, we point out that the resistivity ρ_{xx} in the region $T_{\rm BC} < T < T_{\rm PFS}$ has a different behavior from that in $T_{\rm PFS} < T$ as illustrated in Fig.2. The charge transport in $T_{\rm BC} < T < T_{\rm PFS}$ is dominated by fluxons, which interact with A_{xj} perturbatively. This system resembles with the t-J model of high-T_C cuprates in the anomalous metallic phase, in which holons interact with a dynamical gauge field weakly and the resistivity exhibits the T-linear behavior.[4] In the present case, fluxons have an extra coupling with the CS fluxes generated by fluxons themselves. This should work as an extra resistivity to make ρ_{xx} downwards from the T-linear behavior. It is interesting to observe this behavior in the experiments.

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