Equations for the order parameter and effective magnetic field for nonunitary phases of superfluid Fermi liquids with spin-triplet p-wave pairing

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Abstract

On the basis of generalized Fermi-liquid approach we have derived the systems of coupled equations for the components of the order parameter and effective magnetic field for nonunitary phases of neutral paramagnetic superfluid Fermi liquids with spin-triplet pairing in strong static uniform magnetic field at arbitrary temperatures from the interval $0 \leq T \leq T_c$ (T_c is the normal-superfluid transition temperature). These equations are presented here for dense superfluid neutron matter (existing inside fluid core of neutron stars) with triplet p-wave pairing of $^3He - A_2$ type and with using the effective Skyrme interaction between neutrons.

Key words: superfluidity; Fermi liquid; helium3; neutron matter

1. Introduction

For determination of the phase diagrams of superfluid Fermi liquids (SFLs) with spin-triplet pairing in magnetic field it is necessary to derive equations (Eqs.) for the order parameter (OP) and effective magnetic field (EMF) in these SFLs. Examples of such SFLs are the superfluid phases of ^3He and the superfluid pure neutron matter (SNM) (existing inside liquid core of neutron stars). These SFLs were studied by many authors on the basis of the different methods (see, e.g., monograph [1] and references therein). We have used the generalized Fermi-liquid approach [2] to derive general structure of Eqs. for the OP and EMF, which are valid for arbitrary nonunitary (NU) phases of such SFLs (and SNM) with triplet pairing in sufficiently strong magnetic field in case when pairs have spin $s = 1$ and arbitrary odd angular momentum (see details in [3]). We have specified these Eqs. recently [4] for the case of NU phases of $^3He - A_2$ type for neutral paramagnetic SFLs with p-wave pairing in strong magnetic field at temperatures from the interval $0 \leq T \leq T_c$ with taking into account Landau normal exchange Fermi-liquid amplitudes $F_0^a \neq 0$ and $F_2^a \neq 0$ (in the notations of [1]). (About effect of the Landau parameter F_2^a on the phase transition between unitary A and B phases of superfluid ^3He under strong magnetic field at $T = 0$ see also [5] and references therein.)

In this report we present the obtained Eqs. for the OP and EMF for dense SNM with triplet p-wave pairing of $^3He - A_2$ type in static uniform strong magnetic field at temperatures $0 \leq T \leq T_c$ and use the effective Skyrme interaction as interaction between neutrons (cf., review [6]).

2. SNM with p-wave pairing in magnetic field

As is known [1] the OP for NU phase of $^3He - A_2$ type with p-wave pairing has the form:

$$\Delta_\alpha^{A_2}(\mathbf{p}) = (\Delta_+ \hat{\mathbf{d}}_\alpha + \mathrm{i}\Delta_- \hat{\mathbf{e}}_\alpha)\psi(\hat{\mathbf{p}}), \quad (1)$$

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$$\psi(\hat{\mathbf{p}}) \equiv (\hat{m}_j + i\hat{n}_j)\hat{p}_j, \quad \hat{\mathbf{p}} \equiv \mathbf{p}/p.$$

Here $\Delta_\pm(T) \equiv (\Delta_\uparrow(T) \pm \Delta_\downarrow(T))/2$; $\hat{\mathbf{d}}$ and $\hat{\mathbf{e}}$ are mutually orthogonal real unit vectors in spin space, $\hat{\mathbf{d}} \cdot \hat{\mathbf{e}} = 0$, $\hat{\mathbf{d}}^2 = \hat{\mathbf{e}}^2 = 1$; $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ are mutually orthogonal real unit vectors in orbital space, $\hat{\mathbf{m}} \cdot \hat{\mathbf{n}} = 0$, $\hat{\mathbf{m}}^2 = \hat{\mathbf{n}}^2 = 1$. The value $\eta(\mathbf{p}) \equiv |\mathbf{\Delta}(\mathbf{p}) \times \mathbf{\Delta}^*(\mathbf{p})| \neq 0$ for NU phases of SFL (or SNM).

We have chosen the effective Skyrme interaction as the interaction between neutrons for SNM with spin-triplet p-wave pairing in spatially uniform magnetic field $\mathbf{H}$. The system of coupled Eqs. for the OP of the $^3He - A_2$ type and EMF $\mathbf{H}_{eff}$ inside SNM is simplified in the case of Skyrme interaction because the normal Fermi-liquid Landau's exchange amplitudes $F_l^a \neq 0$ only for $l = 0, 1$. As a result, using general formulas for anomalous and normal distribution functions of quasiparticles [3] for SNM in magnetic field we have derived the set of integral Eqs. for $\xi(p)$ and $\Delta_\uparrow^{A_2}$, $\Delta_\downarrow^{A_2}$. Note that in this case for SNM $\boldsymbol{\xi}(\mathbf{p}) = \xi(p)\mathbf{H}/H = -\mu_n \mathbf{H}_{eff}(p)$ (μ_n is the magnetic dipole moment of neutron) and for $\xi(p)$ we have the following Eq.:

$$\xi(p) = -\mu_n H + (r + sp^2)K_2(\xi) + sK_4(\xi). \qquad (2)$$

Here $r = t_0 + (t_3/6)\rho^\alpha$ and $s = (t_1 - t_2)/(4\hbar^2)$, ρ is density of neutron matter, t_0, t_1, t_2, t_3 and $1/6 \leq \alpha \leq 1/3$ are parameters of the Skyrme interaction (cf., [6], [7]). The functions $K_n(\xi)$ $(n = 2, 4)$ in (2) have the form:

$$K_n(\xi) = \frac{1}{(2\pi)^2\hbar^3} \int_0^{p_{max}} dq q^n \int_0^1 dx \kappa(q, x), \qquad (3)$$

where

$$\kappa(q, x) = \frac{z(q) + \xi(q)}{E_+(q, x^2)} \tanh\Big(\frac{E_+(q, x^2)}{2T}\Big) - \frac{z(q) - \xi(q)}{E_-(q, x^2)} \tanh\Big(\frac{E_-(q, x^2)}{2T}\Big), \qquad (4)$$

$$E_\pm^2(q, x^2) = q^2 \Delta_{\uparrow(\downarrow)}^2 (1 - x^2) + (z(q) \pm \xi(q))^2, \qquad (5)$$

$z(q) = q^2/2m^* - \mu$ (m^* is the effective mass of neutron, μ is the chemical potential). We have taken into account that for SNM with pairing of the $^3He - A_2$ type the OP can be written as $\Delta_{\uparrow(\downarrow)}^{A_2}(T, \xi; q) = q\Delta_{\uparrow(\downarrow)}(T, \xi)$, where functions $\Delta_{\uparrow(\downarrow)}(T, \xi)$ obey the following Eqs.:

$$\Delta_{\uparrow(\downarrow)}(T, \xi) = -\Delta_{\uparrow(\downarrow)}(T, \xi) \frac{c_3}{8\pi^2\hbar^3} \int_0^{p_{max}} dq q^4 \times \int_0^1 dx (1 - x^2) \frac{\tanh\big(\frac{E_\pm(q, x^2)}{2T}\big)}{E_\pm(q, x^2)} \qquad (6)$$

($p_{max} \geq p_F$, p_F is the Fermi momentum). Here $c_3 \equiv t_2(1 + x_2)/\hbar^2 < 0$ is coupling constant of spin-triplet p-wave pairing of neutrons, which is expressed through the parameters t_2 and x_2 of the Skyrme effective interaction (cf. [7]).

3. Conclusions

The system of nonlinear integral Eqs. (2), (6) for the OP and EMF are valid for NU phases of SNM with spin-triplet p-wave pairing of $^3He - A_2$ type in spatially uniform high magnetic field at arbitrary temperatures $0 \leq T \leq T_c$. In general case these Eqs. cannot be solved analytically and it is necessary to use numerical methods for their solving.

Note that these Eqs. for SNM differ from Eqs. for $^3He - A_2$ obtained in our previous paper [8] (where only F_0^a exchange Landau's amplitude was taken into account).

Eqs. (2), (6) for SNM with the Skyrme effective interaction give us the possibility to describe thermodynamics of dense neutron matter in strong magnetic field in superfluid state with triplet pairing existing inside fluid core of neutron stars (cf. [9], [10]). Detailed account of the present study will be reported elsewhere.

Acknowledgements

The author is grateful to S.V. Peletminskii and A.A. Yatsenko for useful discussions. This work was supported by the STCU grant 1480.

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