

# Thermodynamic and magnetic properties of the confined neutral Fermi systems

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## Abstract

The heat capacity and magnetic susceptibility of free Fermi systems confined to spherical pores and to 2D circles are studied theoretically. It is shown that the taking into account the existence of a pore size distribution leads to the smoothing of magnetic susceptibility oscillations. The areal density dependencies of the heat capacity and magnetic susceptibility of the free 2D Fermi gas are obtained.

*Key words:* Neutral Fermi-system; confined geometry; <sup>3</sup>He-<sup>4</sup>He films; magnetic susceptibility

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## 1. Introduction

As it was shown in [1] the geometrical confinement can lead to the appearance some quantum size effects even for free Fermi systems. The examples of the confined Fermi systems are liquid or gaseous <sup>3</sup>He in porous substances (see, for example, [2]), in aerogels [3],[4], the "puddles" of liquid <sup>3</sup>He in <sup>3</sup>He-<sup>4</sup>He thin films [5], <sup>3</sup>He nano-clusters embedded into a <sup>4</sup>He matrix [6], [7] etc. As far as even a free confined Fermi system shows new physical features due to the discreteness of spectrum [1], it seems reasonable to investigate the properties of a neutral free Fermi system being in the confined geometry in more details before considering the more realistic systems such as normal and superfluid liquid <sup>3</sup>He and <sup>3</sup>He-<sup>4</sup>He mixtures.

## 2. The influence of pore size distribution

The oscillations of magnetic susceptibility in dependence on the size of geometrical confinement and particle density have been predicted in [1]. Namely, such

oscillations have to appear, for example, in <sup>3</sup>He-<sup>4</sup>He mixtures confined to spherical pores of radius  $R$  (or cylindrical ones) when one changes the <sup>3</sup>He concentration. But it was unclear whether this effect is observable in experiment or not because some distribution of pore sizes should exist in a real substance (for example, Vycor glass). The Fig. 1 shows the damping of such oscillations at different values of the width of pore size distribution of Gaussian type  $\sigma$ :

$$P(r) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(r - R_0)^2}{2\sigma^2}\right) \quad (1)$$

where  $P(r)$  denotes the probability of the finding the spherical pore with radius  $r$  and  $R_0$  is the mean value of spherical pore radius. It follows also from our calculations that at given values  $R_0$  and  $\sigma$  the oscillations become more pronounced with the temperature lowering (see Fig. 2). Such a feature believes to be caused by the fact that at temperatures well below Fermi temperature  $T_F$  the main contribution to the magnetic susceptibility comes from particles distributed just near Fermi level the position of which depends strongly on the geometrical confinement size and particle density. Note also that relative amplitude of magnetic susceptibility oscillations grows becomes almost constant at temperatures below 0.01 K.

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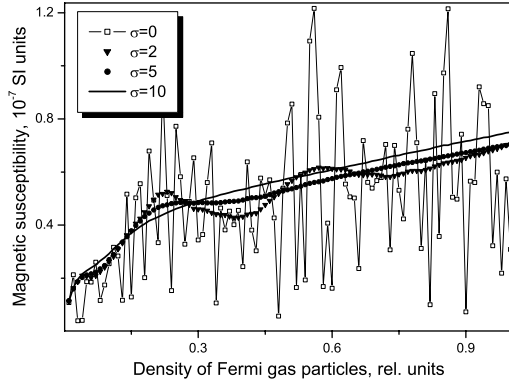


Fig. 1. The damping of magnetic susceptibility oscillations of free Fermi gas in spherical pores with the mean radius  $50 \text{ \AA}$  at temperature  $0.01 \text{ K}$ . The values of distribution width  $\sigma$  are given in  $\text{\AA}$ . The particle density are related to the concentration of normal liquid  $^3\text{He}$  at s.v.p.

### 3. 2D thin films of $^3\text{He}$ - $^4\text{He}$ mixtures

Another system where the quantum size effects can be observed is the 2D thin films of  $^3\text{He}$ - $^4\text{He}$  mixtures. Up to our knowledge it is still unclear which situation takes place in this case - whether the "puddles" of  $^3\text{He}$  exist in the "sea" of  $^4\text{He}$  atoms or the opposite picture occurs. The observation of magnetic susceptibility could shed the light upon this problem.

We have investigated theoretically the heat capacity and magnetic susceptibility of 2D free Fermi gas confined to a circle in the dependence on the circle radius  $R$ , temperature and areal density of particles. The corresponding plots for areal density dependencies at temperature  $0.2 \text{ K}$  are shown in Fig. 3. One can see that again due to quantum size effect there are well pronounced oscillations of magnetic susceptibility in the dependence of areal density. The experimental observation of such oscillations could give the evidence for the existence of  $^3\text{He}$  puddles in 2D thin films of  $^3\text{He}$ - $^4\text{He}$  mixtures. There is also another manifestation of quantum size effects in 2D free Fermi gas. Just near the middle of layer promotion some peaks in heat capacity and magnetic susceptibility are observed. So probably the experimentally observed peaks in the heat capacity of  $^3\text{He}$  multilayers adsorbed on graphite [8] can be attributed in part to the confined geometry influence.

### Acknowledgements

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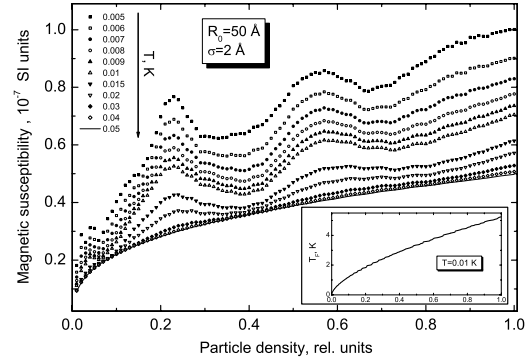


Fig. 2. The damping of magnetic susceptibility oscillations of free Fermi gas for the distribution of spherical pores with the mean radius  $50 \text{ \AA}$  and the width  $\sigma = 2 \text{ \AA}$  at different temperatures. The particle density are related to the concentration of normal liquid  $^3\text{He}$  at s.v.p. The insert shows the particle density dependence of the Fermi energy for spherical pores with radius  $50 \text{ \AA}$  at temperature  $0.01 \text{ K}$ .

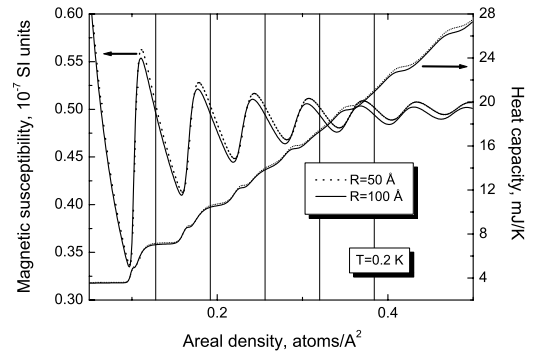


Fig. 3. The magnetic susceptibility oscillations of 2D free Fermi gas confined to the circle with radius  $R$  and its heat capacity versus areal density of particles at temperature  $0.2 \text{ K}$ . The vertical lines correspond to the promotion of layers [8].

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