Energy spectrum of superfluid turbulence made by a quantized vortex tangle without the normal fluid

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Abstract

The energy spectrum of the superfluid turbulence without the normal fluid is studied numerically under the vortex filament model subject to the full nonlocal Biot-Savart law. The almost isotropic and homogeneous vortex tangle is prepared by starting from the Taylor-Green vortex. The vortex tangle freely decaying shows the energy spectrum for $k < 2\pi/\ell$ very similar to the Kolmogorov's -5/3 law, where k is the wave number of the Fourier component of the velocity field and ℓ is the average intervortex spacing.

Key words: superfluid turbulence; vortex tangle; helium4; Kolmogorov law

1. Introduction

Since the pioneering experiments on grid turbulence in superfluid ^{4}He [1], there has been a growing interest in the similarity between superfluid turbulence and classical turbulence [2]. Superfluid turbulence consists of a tangle of quantized vortex filaments. A quantized vortex is a stable topological defect with the definite circulation, free from the decay due to viscous diffusion; these characteristics are quite different from those of classical vortices. Therefore it is not obvious whether superfluid turbulence mimics classical turbulence or not. Vinen studied theoretically this problem, showing that on length scales greater than the average intervortex spacing the superfluid and the normal fluid are coupled together by the mutual friction and behave like a single classical fluid [2]. In order to attack directly the problem without the mutual friction, we study the energy spectrum of the quantized vortex tangle without the normal fluid, in connection with the Kolmogorov's -5/3 law that is the most important statistical law in classical turbulence@[3].

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2. Vortex filament and the energy spectrum

The vortex dynamics is calculated numerically under the full nonlocal Biot-Savart law; the detail of the vortex dynamics and the numerical scheme is described in our previous paper [4]. A vortex filament is represented by the parametric form $\boldsymbol{s} = \boldsymbol{s}(\xi, t)$, where $\boldsymbol{s}$ refers to a point on the filament, the prime denotes differentiation with respect to the arc length ξ. The energy spectrum $E(k)$ is defined as $E = \int_0^\infty dk E(k)$, where E is the total kinetic energy per unit volume and k is the wave number of the velocity field. The energy spectrum of the superflow made by the vortex filaments is given by [5]

$$E(k) = \frac{\rho_s \kappa^2}{2(2\pi)^3} \int \frac{d\Omega_k}{|\boldsymbol{k}|^2} \int\int d\xi_1 d\xi_2 \times \boldsymbol{s}'(\xi_1) \cdot \boldsymbol{s}'(\xi_2) e^{-i\boldsymbol{k}\cdot(\boldsymbol{s}(\xi_1)-\boldsymbol{s}(\xi_2))}, \quad (1)$$

where $d\Omega_k$ denotes the volume element $k^2 \sin\theta_k d\theta_k d\phi_k$ in spherical coordinates and the double integration is taken along the filaments. The energy spectrum $E(k)$ is calculated for the vortex configuration $\boldsymbol{s}(\xi)$ obtained by the simulation of the dynamics.

3. Kolmogorov's law

Figure 1 shows the decay of the vortex tangle without mutual friction, starting from the Taylor-Green vortex. All boundaries are assumed to be smooth solid walls. Although the initial vortices are highly polarized, they become an almost homogeneous and isotropic tangle through the chaotic dynamics including lots of reconnections. The self-similar continuous cascade process by which vortices break up to smaller ones through reconnections decays the tangle. The vortices whose size is smaller than the numerical space resolution $\Delta\xi$ are eliminated in our calculation; this cutoff procedure can be justified because of the decay rate almost independent of the cutoff scale [4].

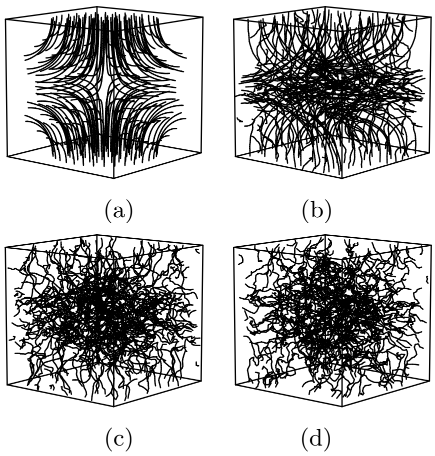

Fig. 1. Time evolution of the vortex tangle at $t = 0$ sec(a), 30.0 sec(b), 50.0 sec(c) and 70.0 sec(d). The system is a 1cm^3 cube.

The energy spectrum changes with the vortex configuration. Although the spectrum is initially affected by the Taylor-Green configuration, it converges to a characteristic one as the tangle loses the memory of the initial configuration. This convergence is found also in the energy dissipation rate $\epsilon = -dE/dt$ due to the cutoff procedure; the change of ϵ becomes slow free from the artifact of the early large dissipation.

The converged energy spectrum at $t = 70$ sec is shown in Fig.2. The slope of the spectrum is changed about at $k = 2\pi/\ell$, where ℓ is the average intervortex spacing. The spectrum for $k > 2\pi/\ell$ has k^{-1} behavior which comes from the velocity field near each vortex line. On the contrary, the spectrum for $k < 2\pi/\ell$ directly reflects the turbulent vortex configuration to have the Kolmogorov form $k^{-5/3}$. We can show that the obtained energy spectrum agrees quantitatively with the Kolmogorov law $E(k) = C\epsilon^{2/3}k^{-5/3}$. Since the Kolmogorov constant C is known as the parameter of order unity, we use $C = 1$ here. The energy dissipation rate is found to be $\epsilon = 1.866 \times 10^{-7}\text{erg}\cdot\text{sec}^{-1}$ in our tangle at $t = 70$sec. Thus the Kolmogorov spectrum is determined uniquely, and our energy spectrum for $k < 2\pi/\ell$ is consistent with the Kolmogorov law not only on the wave number dependence but also on the absolute value. Our tangle allows the dissipative mechanism due to the cutoff to work only at the largest wave number $k \sim 2\pi/\Delta\xi = 343\text{cm}^{-1}$. Nevertheless the energy spectrum at small k is determined by that dissipation rate. This result supports just the picture of the inertial range.

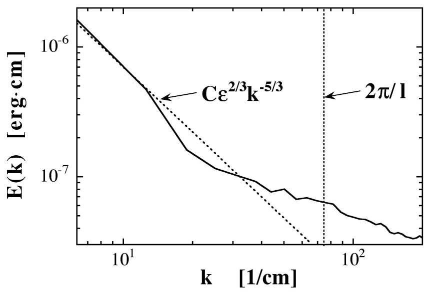

Fig. 2. Comparison of the energy spectrum (solid line) at $t = 70$ sec with the Kolmogorov law $E(k) = C\epsilon^{2/3}k^{-5/3}$ (dotted line) with $C = 1$ and $\epsilon = 1.866 \times 10^{-7}$ erg $\cdot$ sec^{-1}.

4. Conclusion

A tangle of quantized vortex filaments makes the energy spectrum for low k region consistent quantitatively with the Kolmogorov law, without the normal fluid. The detailed studies, such as Kelvin wave cascade in high k region and the relation with the vortex size distribution, are now in progress.

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