System-Size Dependences and Correlation Effects of Josephson current through One-Dimensional Josephson networks

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Abstract

D.C. Josephson effects are studied in Josephson junction arrays with *finite* number of junctions. We study how the Coulomb interaction affects the maximum Josephson current. The tendency of the charge order is also discussed by studying the phase-current relations.

Key words: Josephson junction array; Coulomb blocakde; charge order

The series of small Josephson junctions enables us to study the repulsive interaction of the Cooper pairs between different islands. In several theoretical issues, the phase diagram of such systems have been studied [1–3]. Experimentally, however, the coherent tunneling through the Josephson junction arrays are hindered by the presence of impurity charges in substrates and oxides of tunnel junction. Recently, transport properties of one-dimensional Josephson junction arrays consisting of six islands have been measured by Oudenaarden and Mooij [4]. They have removed the impurity charge effects by tuning six gate voltages applied to each island, and have observed the transition to insulator when the filling of Cooper-pairs per island is controlled near an integer or half-integer by the gate voltage.

In this paper, the Josephson junction arrays consisting of finite number of the islands shown in Fig. 1 are studied. Each island couples to its neighboring islands with a capacitance C and a Josephson energy $E_{\rm J}$. The Josephson junction array is connected to two large superconducting leads through a capacitance C and a Josephson energy $E_{\rm J}$. The gate voltage $V_{\rm g}$ is applied to all islands through a gate capacitance C_0.

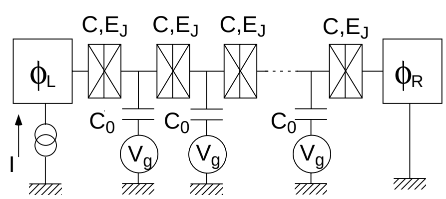

Fig. 1. Circuit considered in this paper.

For simplicity, we assume $C_0 \gg C$. The effective Hamiltonian is obtained near the half filling ($\bar{n} = C_0 V_{\rm g}/2e = 1/2$) as

$$H = \sum_{i=1}^{N-1} V \left(n_i - \frac{1}{2}\right)\left(n_{i+1} - \frac{1}{2}\right) - \mu \sum_{i=1}^{N} n_i,$$

$$- \frac{E_{\rm J}}{2} \sum_{i=1}^{N-1} \left(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i\right)$$

$$- \frac{E_{\rm J}}{2}\left(b_1 + b_1^\dagger\right) - \frac{E_{\rm J}}{2}\left(b_N e^{-{\rm i}\phi} + b_N^\dagger e^{{\rm i}\phi}\right), \quad (1)$$

where $V = (2e)^2/2C_0 \times 2C/C_0$ is the nearesr-neighbor Cpilomb interaction, b_is ($b_i^\dagger$s) are anhilation (creation) operators of Cooper pairs satisfying the hardcore boson condition $b_i^2 = 0$, and $n_i = b_i^\dagger b_i$. The chemical potential μ can be controlled by the gate voltage, and the phase

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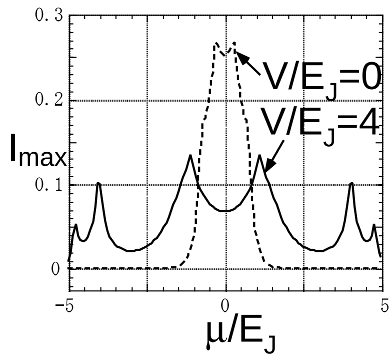

Fig. 2. Maximum supercurrent is shown as a function of the chemical potential μ. The solid and dashed curves correspond to $V/E_{\rm J} = 4$ and $V/E_{\rm J} = 0$, respectively.

difference is denoted with $\phi = \phi_{\rm L} - \phi_{\rm R}$.

Below, we focus on the six-island systems ($N = 6$). The supercurrent at zero temperature is calculated by $I = \partial E_0/\partial\phi$, where $E_0(\phi)$ is a ground state energy. The maximum supercurrent is evaluated as $I_{\max} = \max[I(\phi)]$. In Fig. 2, the maximum current $I_{\max}$ is shown as a function of the chemical potential μ. The solid and dashed curves correspond to $V/E_{\rm J} = 4$ and $V/E_{\rm J} = 0$, respectively. The maximum current has six peaks for $V/E_{\rm J} = 0$. When the nearest-neighbor interaction is introduced, these peaks move away from the center $\mu = 0$, and the peak heights are suppressed.

In Fig. 3, the supercurrent I is shown as a function of ϕ at several values of μ denoted in the inset. We first discuss the result of $V/E_{\rm J} = 1$ shown in Fig. 3 (a). When the chemical potential μ is taken at the resonant peak points (the points A, C, and E in the inset), the current has a maximum at $\phi = \pi$. Even when μ is taken in the off-resonant region (the points B and D), the current has a maximum near $\phi = \pi$. Thus, the resonant peaks in the maximum current are decided by the behavior near $\phi = \pi$ in the phase-current relation. On the other hand, the phase-current relation near $\phi = 0$ behaves differently. We study the slope $Y = \frac{\partial I}{\partial \phi}\Big|_{\phi=0}$ which corresponds to the helicity modulus. As seen in Fig. 3 (a), this quantity decreases monotonously as $|\mu|$ increases, and has no resonant peaks in contrast to the maximum current $I_{\max}$. In other words, the helicity modulus is independent of the maximum current, and depends weakly on the system size for small $V/E_{\rm J}$.

The result of $V/E_{\rm J} = 4$ is shown in Fig. 3 (b). In this case, the helicity modulus Y does not change monotonically, and becomes dependent on the maximum current $I_{\max}$. In other words, the helicity modulus becomes sensitive to the finite-size effect as well as $I_{\max}$. This change in the phase-current relation for large $V/E_{\rm J}$ is expected to come from the tendency of the charge order.

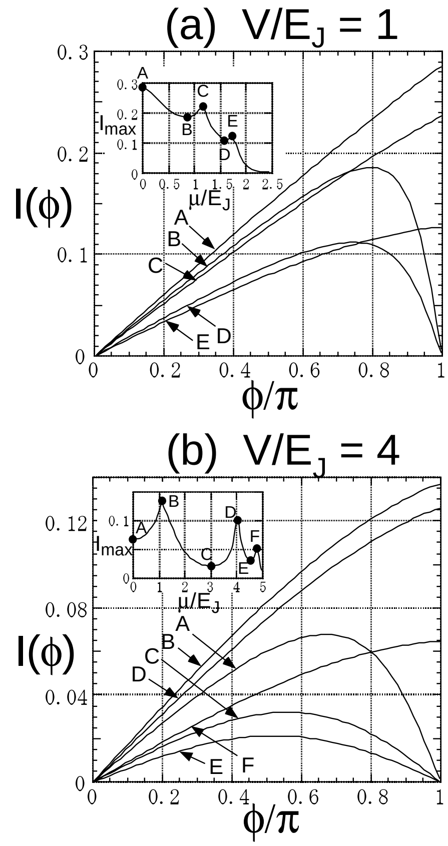

Fig. 3. Phase-current relation (a) for $V/E_{\rm J} = 1$ and (b) for $V/E_{\rm J} = 4$. The values of μ are chosen as on- and off-resonant points as shown in the inset, which shows the maximum current as a function of μ.

In summary, the interaction effects on supercurrent through the Josephson junction arrays consists of finite number of islands have been studied. Due to the finite size effect, the maximum current has several peaks as the gate voltage is changed. For small $V/E_{\rm J}$ the helicity modulus Y changes monotonically as $|\mu|$ changes, while for large $V/E_{\rm J}$ peak structures appear in Y. We hope that these behaviors are observed in real experiments.

References

[1] C. Bruder, R. Fazio, and G. Schön, Phys. Rev. B 47 (1993) 342.

[2] A. A. Odintsov, Phys. Rev. B 54 (1996) 1228.

[3] L. I. Glazman, and A. Larkin, Phys. Rev. Lett. 79 (1997) 3736.

[4] A. van Oudenaarden, and J. E. Mooij, *Quantum vortices and quantum interference effects in circuits of small tunnel junctions*, chap. 8 (Delft University Press, Netherlands, 1998).