Thermodynamics of metastable superfluid helium

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Abstract

Using Landau's theory of elementary excitations together with a simple model for the interactions between excitations, we calculate the thermodynamic properties of superfluid helium-4 at pressures below the saturated vapor pressure and above the normal freezing temperature.

Key words: Liquid helium-4; superfluidity; phase diagram; lambda line

In the last few years, several experimental investigations of the properties of metastable liquid helium have been reported [1]. Studies have been made of liquid helium at pressures below the saturated vapor pressure and, more recently, the properties above the normal freezing pressure P_{f} have also been investigated [3]. In this paper we report on calculations of the thermodynamic functions of metastable superfluid helium.

The calculations are based on Landau's theory [4] in which thermodynamic quantities are expressed in terms of sums over the distribution of thermally-excited elementary excitations. At low temperatures where the number density of excitations is small, the interactions between excitations can be neglected and the energy of an excitation of wave number q is ϵ_q^0, independent of temperature. Under these conditions the normal fluid density ρ_n, for example, is given by:

$$\rho_n = \frac{\hbar^2}{3kT}\sum_q \overline{n}_q^0(\overline{n}_q^0+1)q^2, \qquad (1)$$

where $\overline{n}_q^0 \equiv (\exp\epsilon_q^0/kT-1)^{-1}$ is the Bose-Einstein distribution function. For the dispersion curve of the elementary excitations, we use the results from the density-functional theory of Dalfovo *et al.* [5]. When the density of excitations becomes large, it is necessary to include some form of interaction between the excitations. Here, we use a modified form of the roton liquid theory [2] in which the effective energy of an excitation is taken to be corrected by an amount which is proportional to the sum of the energy (without interactions) of all other excitations. In analogy with Fermi liquid theory, we expand the interaction in terms of Legendre polynomials of the angle $\theta_{\mathbf{qq'}}$ between the momenta of the interacting excitations $\mathbf{q}$ and $\mathbf{q}'$. Only f_0 and f_1 affect the thermodynamic functions. In the pressure range over which liquid helium is stable, we estimate the value of f_0 and f_1 by requiring that the theory give the correct entropy along the λ-line and the correct λ-temperature. We then extrapolate f_0 and f_1 into the range of density in which the liquid is metastable. Details of the calculation will be published separately [6].

The calculated phase diagram at low pressure is shown in Fig. 1. We have included in Fig. 1 the location of the spinodal for the liquid in the normal phase as estimated previously by Hall and Maris [7]. Our results indicate that the spinodal meets the λ-line at a temperature of 2.18 K and a pressure of around -7.3 bars. We also show the line in the P-T plane along which the thermal expansion coefficient α is zero. This curve reaches the λ-line tangentially at around -5 bars. Included in Fig. 1 are a set of lines of constant entropy.

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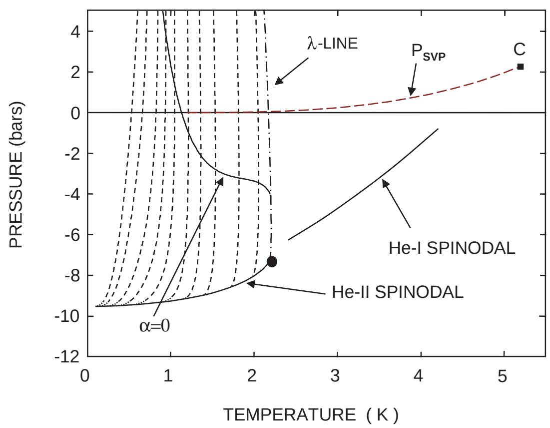

Fig. 1. The calculated phase diagram of liquid helium showing the location of the λ-line, the spinodal, and the line where the thermal expansion coefficient α is zero. The entropy is constant along each of the dashed curves. The intersection of the λ-line with the spinodal is indicated by a solid circle.

These lines are important because they determine the P-T trajectory that is followed during an expansion of the liquid in a cavitation experiment.

In Fig. 2 we show the calculated variation of T_λ and the entropy S_λ with pressure. The variation of T_λ with pressure arises from two competing effects: the decrease in the roton gap with increasing P makes T_λ decrease, while on the other hand the increase in ρ means that ρ_n becomes equal to ρ at a higher temperature, thus raising T_λ. It can be seen that the variation of the entropy along the λ-line is quite modest; our prediction is that the entropy at T_λ increases from 1.56 J g^{-1} K^{-1} at the saturated vapor pressure to 2.00 J g^{-1} K^{-1} at -7 bars. As a result of this small variation, the λ-line runs roughly parallel to the adjacent lines of constant S (see Fig. 1).

The theory can also be used to estimate the behavior of helium at pressures above the normal freezing pressure P_f. However, we do not have a reliable way to estimate the behavior of the roton gap at high pressures. If we evaluate the excitation dispersion curve using the Dalfovo *et al.* theory [5], we find that $\Delta \rightarrow 0$ at around 200 bars. At this pressure $P_\mathrm{c}(0)$, the softening of the roton mode might lead to crystal formation. But there is no way to be sure that the theory remains accurate up to this pressure. However we can consider the temperature boundary of the superfluid state at a pressure P above P_f. Two possibilities come to mind: (1) if $f_0(P)$ is not too negative, the superfluid to normal transition will take place on the extension of the λ-line above P_f; (2) if $f_0(P)$ becomes sufficiently negative, a critical temperature $T_\mathrm{c}(P)$ at which the roton liquid presents a Pomeranchuk instability as described in [2] will be reached before the λ-transition occurs. At the present, the only way we know how to estimate f_0 and f_1 is to perform an extrapolation of values obtained in the pressure range where helium is stable. The results that can be obtained in this way will be described in a future publication [8].

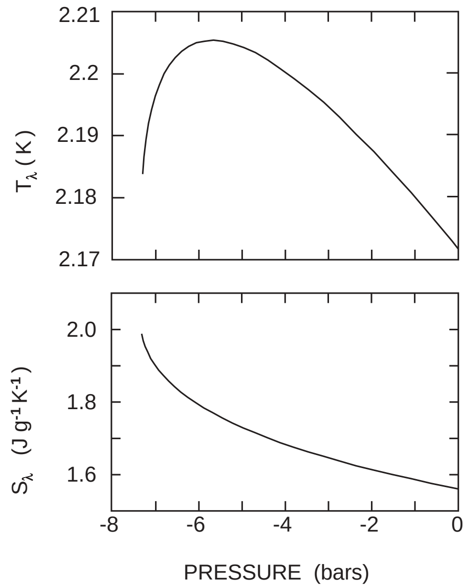

Fig. 2. The calculated λ-temperature and entropy as a function of pressure.

Acknowledgements

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