Transverse sound in aerogel with liquid ^{4}He

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Abstract

An experiment was performed to measure transverse sound resonances in a square slab of aerogel filled with liquid ^{4}He. Resonances have been observed both in the superfluid and normal phase. The dynamics of the system was modeled by combining the equations of two-fluid hydrodynamics of helium with those of elasticity of aerogel.

Key words: transverse sound; superfluidity; aerogel.

1. Introduction

The dynamics of elastic porous materials saturated with superfluid helium combines properties of elastic solid and superfluid liquid. Because of viscous locking, the normal component of helium is oscillating together with the porous matrix, while the superfluid fraction can flow without dissipation. Four sound modes can thus propagate: two longitudinal and two transverse [1]. Very open aerogels are especially interesting, because their density and compressibility can be smaller than those of helium. The fast and slow longitudinal modes have been observed and quantified for ^{4}He in aerogel [2] and used to measure the superfluid fraction of ^{3}He in aerogel [3]. The transverse sound in ^{4}He was also observed by pulsed technique [4], but never fully studied.

2. The experiment

The transverse sound resonances were measured in a square slab (side $D = 18$ mm; thickness $L = 2$ mm) of silica aerogel of porosity $\phi = 0.91$ filled with liquid ^{4}He at saturated vapour pressure. Two identical piezoceramic shear plates, acting as driver and detector, were glued to the two sides of the slab (see Fig. 1). Each plate was cut into two rectangular halves which could be driven or monitored either in- or out-of-phase, hence enabling selective excitation and detection of vibrational modes of different symmetry.

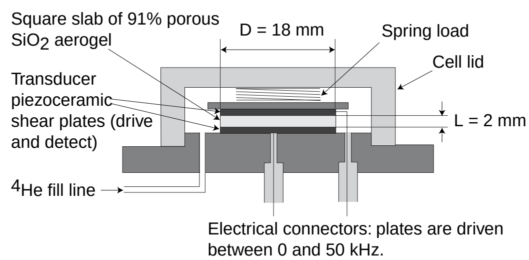

Fig. 1. Experimental cell.

3. Modelling the system

The presence of the aerogel requires modification of the equations of motion of the two-fluid model (Eqs. 1-5). The aerogel adds its mass, ρ_a, to the normal component, ρ_n, and provides an extra restoring force due to its elasticity. Explicit shear and compression terms have been added to the equation of motion of the normal component (Eq. 4). A thin inert layer of helium (of overall density ρ_o and volume fraction ϕ_0) covers the aerogel, latching on its oscillation. The superfluid component is partly coupled to the aerogel motion because of the tortuosity of its strands. The extra coupled mass is $\rho_s\chi$ [1] where ρ_s is the superfluid density and χ is the geometric "drag factor" (Eqs. 3 and 4). The displacement of the aerogel with the normal component is denoted by $\mathbf{u_n}$, and that of the superfluid component

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by $\mathbf{u_s}$. We define $\rho = \phi' \rho_{\rm bulk}$, where $\phi' = \phi - \phi_0$ is the aerogel porosity corrected for the volume occupied by the "inert layer". λ_a and μ_a are the Lamé coefficients of the aerogel. The terms containing entropy and temperature are small and have been neglected.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_n \frac{\partial \mathbf{u}_n}{\partial t} + \rho_s \frac{\partial \mathbf{u}_s}{\partial t}) = 0 \quad (1)$$

$$\frac{\partial \rho_a}{\partial t} + \nabla \cdot (\rho_a \frac{\partial \mathbf{u}_n}{\partial t}) = 0 \quad (2)$$

$$\rho_s \frac{\partial^2 \mathbf{u}_s}{\partial t^2} - (\frac{\chi}{1-\chi}) \rho_s (\frac{\partial^2 \mathbf{u}_n}{\partial t^2} - \frac{\partial^2 \mathbf{u}_s}{\partial t^2}) = -\frac{\rho_s}{\rho} \nabla P \quad (3)$$

$$(\rho_a + \rho_o + \rho_n) \frac{\partial^2 \mathbf{u}_n}{\partial t^2} + (\frac{\chi}{1-\chi}) \rho_s (\frac{\partial^2 \mathbf{u}_n}{\partial t^2} - \frac{\partial^2 \mathbf{u}_s}{\partial t^2}) = -\frac{\rho_n}{\rho} \nabla P + (\lambda_a + \mu_a) \nabla (\nabla \cdot \mathbf{u}_n) + \mu_a \nabla^2 \mathbf{u}_n \quad (4)$$

$$\nabla \times \frac{\partial \mathbf{u}_s}{\partial t} = 0 \quad (5)$$

The boundary conditions for the aerogel in our resonator are fixed at the aerogel flat surfaces and free at its perimeter. Eqs. 1-5 were solved for two types of boundary conditions: a resonator with infinitely wide parallel plates spaced L apart, and a resonator represented by a square 'box' with rigid boundaries of sides D and thickness L. The spatial coordinates were chosen to be $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ along the two sides of the slab and $\hat{\mathbf{z}}$ along its thickness. Both $\mathbf{u_n}$ and $\mathbf{u_s}$ were restricted to be along the $\hat{\mathbf{x}}$ axis. The plane wave solution (propagating in the direction $\hat{\mathbf{z}}$) for an infinitely wide slab has the following eigenfrequency:

$$f = \frac{n_z C_{ta}}{2L} \left(\frac{\rho_a}{\rho_a + \rho_n + \rho_o + \chi \rho_s} \right)^{1/2}, \quad (6)$$

where C_{ta} is the velocity of sound in empty aerogel and $n_z = 1, 2, 3...$ is the number of antinodes in the mode. For the standing waves in the box, both $\mathbf{u_n}$ and $\mathbf{u_s}$ preserve dependence on the three spatial coordinates, subject to the following boundary conditions: $\mathbf{u_n}$ is taken to be zero everywhere on the slab faces, whilst $\mathbf{u_s}$ is allowed to have non-zero component parallel to the faces. Each mode of vibration is described by the set of integers $(n_x n_y n_z, s_x s_y s_z)$, being the number of antinodes along the indicated direction for the normal component with the aerogel, and the superfluid component. However, the condition of irrotational superfluid (Eq. 5) requires that s_y and s_z be zero.

4. Results

Fig. 2 shows the experimental frequencies along with those calculated with the model. The experimental data include the fundamental resonance of the empty aerogel (nearly temperature independent) and the resonance of the aerogel saturated with liquid ^{4}He. One notices the temperature independent behaviour in the normal phase and the progressive increase in the frequency of the mode as the superfluid decouples from the oscillation. However, when helium is all superfluid ($T < 1$ K) the frequency remains lower than that one of the empty aerogel, due to the extra coupling provided by tortuosity. For the calculation we used the

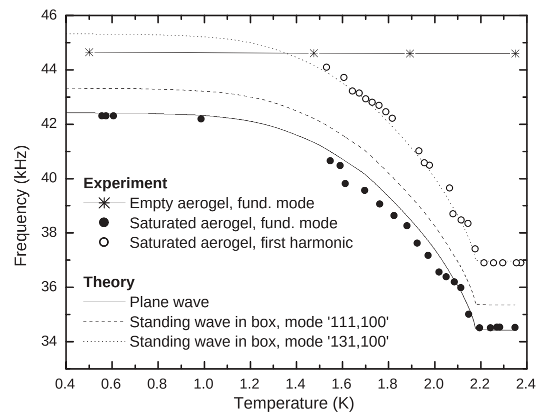

Fig. 2. Comparison between experimental results and theory.

nominal density of aerogel $\rho_a = 0.200$ g/cm^3, the inert layer overall density $\rho_o = 0.010$ g/cm^3 and its volume fraction $\phi_o = 0.05$, the overall volume fraction of the liquid part $\phi' = 0.86$, the drag factor $\chi = 0.093$ obtained from [6] and modified for the presence of the solid inert layer. The velocity of transverse sound in empty aerogel was measured to be $C_{ta} = 178.5$ m/s and found to be nearly temperature independent. In Fig. 2 the solid line corresponds to the solution for the case of a resonator with infinitely wide parallel plates (pure transverse sound), while the dashed line is the fundamental mode for the case of a standing wave in a box resonator with fixed boundaries. Mode $'131, 100'$ is the next mode detectable by the transducers when the semi-plates are operated in phase. The curves reproduce well the trend of the experimental data.

Acknowledgements

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