

# Interplay of Ferromagnetism and Superconductivity: Domain Structure

Edouard B. Sonin <sup>1</sup>,

*Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem 91904, Israel*

---

## Abstract

In superconducting ferromagnets, where superconductivity and ferromagnetism coexist at the same bulk, the equilibrium domain structure is absent in the Meissner state, but it does exist in the spontaneous vortex phase. In a superconductor-ferromagnet bilayer superconductivity and ferromagnetism are separated in space, but strongly affect each other. The superconducting layer shrinks the size of domains in the ferromagnetic layer by a numerical factor, in contrast to superconducting ferromagnets, where superconductivity can strongly increase the domain size.

*Key words:* Josephson effect; flux lines; YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>; specific heat

---

Coexistence of superconductivity and ferromagnetism has been revealed experimentally in various materials. Up to now the theory mostly addressed macroscopically uniform structures [1], whereas ferromagnetic materials, even ideally uniform ones, inevitably have a domain structure. The present work addresses the domain structure in superconducting ferromagnets, where superconductivity and ferromagnetism coexist at the same bulk, and in a superconductor-ferromagnet bilayer, where superconductivity and ferromagnetism are separated in space, but mutually affect each other via long-range magnetic fields.

In order to find the distribution of the magnetic field  $\mathbf{H}$  and the magnetic induction  $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$  ( $\mathbf{M}$  is the spontaneous magnetization) one should solve the equations of magnetostatics and London electrodynamics. The magnetic induction  $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$  is divergence-free,  $\nabla \cdot \mathbf{B} = 0$ , but the magnetic field  $\mathbf{H}$  is not:  $\nabla \cdot \mathbf{H} = 4\pi\rho_M$ , where  $\rho_M = -\nabla \cdot \mathbf{M}$  is the magnetic charge.

We consider a domain structure in standard geometry [2]: a slab of the thickness  $d$  along the anisotropy easy axis and infinite in other directions. The magnetic charges appear only at boundaries of the slab (Fig. 1). Without an external magnetic field in a single-domain

structure (Fig. 1a),  $\mathbf{B} = 0$  and there exists an uniform magnetostatic field  $\mathbf{H} = -4\pi\mathbf{M}$  and high magnetostatic energy  $H^2/8\pi \sim M^2$  in the entire sample. For a stripe domain structure one can find the exact solution for  $\mathbf{H}$  using the method of complex variables [3]:

$$-H_x + iH_y = 4M \left[ \ln \tan \frac{\pi w}{2l} - \ln \tan \frac{\pi(w - id)}{2l} \right]. \quad (1)$$

where  $w = x + iy$ . Inside the domain bulk  $\mathbf{H} \approx 0$ , except for the area  $\sim l^2$  near the slab boundary (Fig. 1b). The equilibrium period  $l$  ( $l \ll d$ ) is determined by minimization of the sum of the magnetostatic energy and the energy of domain walls [2]:

$$l = \sqrt{\frac{\alpha K}{M^2}} \delta d, \quad (2)$$

where  $K$  is the anisotropy energy,  $\delta$  is the wall thickness, and  $\alpha$  is a numerical factor.

In a superconducting ferromagnet in the Meissner state  $\mathbf{B}$  must vanish in the bulk. This is compatible only with a single-domain structure, and *the equilibrium domain structure is impossible in the Meissner state*. Domain walls may appear only in metastable states. If  $4\pi M > H_{c1}$ , the Meissner state is absent, and the superconducting ferromagnet is in the mixed state

---

<sup>1</sup> Corresponding author. E-mail: sonin@cc.huji.ac.il

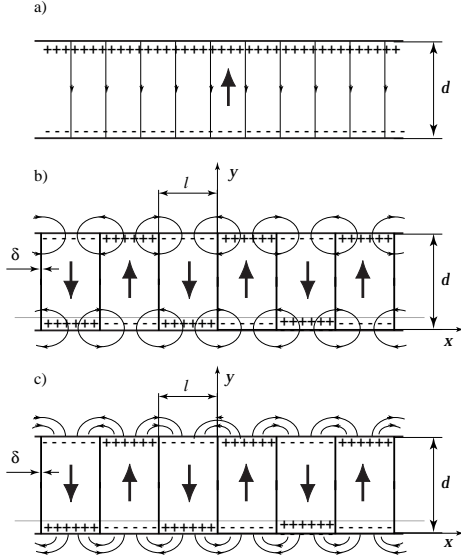


Fig. 1. Domain structure in normal and superconducting ferromagnets. The thick arrows show directions of the magnetic moment  $\mathbf{M}$ , the thin lines with arrows are force lines of the magnetostatic field  $\mathbf{H}$ . The magnetic charges are shown by + and -. a) A single-domain structure. In the whole bulk  $B = 0$  and  $\mathbf{H} = -4\pi\mathbf{M}$ . b) A stripe domain structure in a normal ferromagnet. The magnetostatic fields are present in areas  $\sim l^2$  inside and outside the sample. In the rest parts of domains  $H = 0$  and  $\mathbf{B} = 4\pi\mathbf{M}$ . c) A superconducting ferromagnet in the spontaneous vortex phase with a rigid vortex array. The magnetostatic fields appear only in areas  $\sim l^2$  outside the sample. In the bulk of domains  $H = 0$  and  $B = B_0(4\pi M)$ .

even in zero external field ( $H = 0$ ). This is the *spontaneous vortex phase* with nonzero magnetic induction  $B = B_0(4\pi M)$  in the bulk [1]. Here  $H_{c1}$  is the lower critical field and  $B_0(H)$  is the equilibrium magnetization curve for a nonmagnetic type II superconductor.

Solving the magnetostatic problem for the spontaneous vortex phase yields again Eq. (2) for the period  $l$ , but with  $M$  replaced by the effective magnetization  $\tilde{M} = B_0(4\pi M)/4\pi$  [4]. If  $4\pi M \gg H_{c1}$ , then  $\tilde{M} \rightarrow M$  and the effect of superconductivity on the domain structure vanishes. If  $4\pi M \rightarrow H_{c1}$ ,  $\tilde{M}$  vanishes and the period  $l$  becomes infinite, as it should be in the Meissner state  $4\pi M < H_{c1}$ . Up to now we assumed that magnetostatic field penetrates into a superconducting ferromagnet like into a normal ferromagnet. But for a rigid or strongly pinned vortex array, the magnetostatic fields penetrate only into the layer of thickness  $\lambda$ , and penetration is insignificant if  $\lambda \ll l$  (Fig. 1c). This increases the magnetic field outside the sample, as well as the total magnetostatic energy, by a factor of 2, while the correspondingly period  $l$  decreases by a factor of  $\sqrt{2}$ , in analogy with the effect of a superconducting layer on a domain size in a ferromagnetic layer, which is discussed below.

If the ferromagnetic film is put on a superconduct-

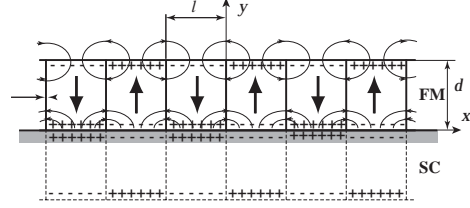


Fig. 2. Magnetic charges (+ and -) and magnetic flux (thin lines with arrows) in a ferromagnetic film (FM) on a superconducting slab (SC).

ing slab with the London penetration depth much less than the domain size  $l$  and the film thickness  $d$ , penetration of the magnetic field into the superconductor is insignificant, and one obtains the exact solution with help of the image charges in the superconductor [3]:

$$-H_x + iH_y = 4M \left[ 2 \ln \tan \frac{\pi w}{2l} - \ln \tan \frac{\pi(w - id)}{2l} - \ln \tan \frac{\pi(w + id)}{2l} \right]. \quad (3)$$

If  $d \gg l$ , the magnetostatic fields on two boundaries of the ferromagnet ( $y = 0$  and  $y = d$ ) do not overlap and the superconducting layer affects only the field distribution near  $y = 0$ :  $\mathbf{H} = 0$  at  $y < 0$ , but at  $y < 0$   $\mathbf{H} = 0$ , is by a factor 2 larger. As a result, the magnetostatic energy near  $y = 0$  is by a factor 2 larger. Eventually the superconducting layer shrinks domains in a neighboring ferromagnetic layer by a factor  $\sqrt{1.5}$ .

In summary, in a superconducting ferromagnet the equilibrium domain structure is absent in the Meissner state, but exists in the spontaneous vortex phase, though with the period of the domain structure may essentially exceed that in the normal ferromagnet. In contrast, in a superconductor-ferromagnet bilayer, in which superconductivity and ferromagnetism are separated in space, superconductivity shrinks domains in the ferromagnetic layer by a numerical factor.

## Acknowledgements

The work has been supported by the grant of the Israel Academy of Sciences and Humanities.

## References

- [1] E.B. Sonin, I. Felner, Phys. Rev. B **57**, R14000 (1998).
- [2] L.D. Landau, E.M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1984).
- [3] E.B. Sonin, cond-mat/0102102.
- [4] E.B. Sonin, cond-mat/0202193.