Universal features in ensembles of small superconducting grains

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Abstract

The physical properties of an ensemble of disordered ultra-small superconducting grains are analyzed by means of a simple Hamiltonian. We find that both the thermodynamical observables and the interplay between the pairing and exchange interactions display universal features.

Key words: Superconducting grains; Exchange interaction; Mesoscopic fluctuations

In an ensemble of ultrasmall metallic dots the disorder plays a key role due to the unavoidable different shapes of the grains and the presence of the impurities. This disorder will lead to mesoscopic fluctuations of the energy levels. Nevertheless the pairing and exchange interactions between the electrons can lead to ordered phases with universal properties, which we study in this work. In an isolated mesoscopic grain the electrons are ruled by a fairly general Hamiltonian [1] in the limit of very large dimensionless conductance:

$$\hat{H}_{\rm univ} = \hat{H}_{\rm kin} - J\hat{\boldsymbol{S}}^2 - \lambda\, \hat{T}^\dagger\, \hat{T}. \qquad (1)$$

Here, the kinetic term reads $\sum_{\alpha,\sigma} \epsilon_\alpha\, \hat{c}^\dagger_{\alpha,\sigma} \hat{c}_{\alpha,\sigma}$, the total spin operator is $\frac{1}{2}\sum_{\alpha,\sigma_1\sigma_2} \boldsymbol{\sigma}_{\sigma_1\sigma_2} \hat{c}^\dagger_{\alpha,\sigma_1} \hat{c}_{\alpha,\sigma_2}$ and the pair scattering operator $\hat{T} = \sum_\alpha c_{\alpha,\downarrow} c_{\alpha,\uparrow}$, where α spans a shell of Ω pairs of single particle energy levels ϵ_α, with the annihilation operator $c_{\alpha,\sigma}$ and the Pauli matrices $\sigma^i_{\sigma_1\sigma_2}$. The energies ϵ_α are distributed according to the Gaussian Orthogonal Ensemble (GOE), which describes the case of time-reversal symmetry with no spin-orbit coupling [2].

Thermodynamics: Let us first consider the case of negligible exchange interaction ($J = 0$). This model has been extensively used to investigate superconducting correlations in small metallic grains where the mean level spacing $\delta \sim 1/(N(0)V)$ may be comparable or even larger than the BCS energy scale $\Delta = \omega_D/2\sinh(\delta/\lambda)$, being $\omega_D = \Omega\delta$ the Debye energy. Experiments [3] by Ralph, Black and Tinkham revealed a gap related to superconductivity in individual nanosized metallic grains. The characterization of "superconductivity" from the bulk to the nm regime has been studied in recent theoretical works [4,5]. It is remarkable that pairing determines strong fluctuational "superconductivity" even in ultrasmall grains, where $\delta \gtrsim \Delta$ [4]. In this section we present results for ensembles of ultrasmall grains at finite temperature. Our work is motivated by the fact that the thermodynamics is a unique experimental tool in detecting unambiguous traces of superconducting correlations in the region $\delta \gtrsim \Delta$ [6], where tunneling spectroscopy on *individual* dots is not sensitive enough. A simple scaling theory [7] can capture the physics of the system at very low temperature where the thermodynamics is determined by samples with the closest level spacing to the Fermi energy much smaller than the average δ. In the regime $T, \Delta \ll \delta$ only few levels around the Fermi energy are important and the system can be rescaled to a smaller one $\Omega' \ll \Omega$, with renormalized coupling constant $\lambda \equiv \tilde{\lambda}_\Omega \to \tilde{\lambda}_{\Omega'}$ [4]. By means of this procedure we evaluate analytically the expression for different thermodynamic quantities of an ensemble of

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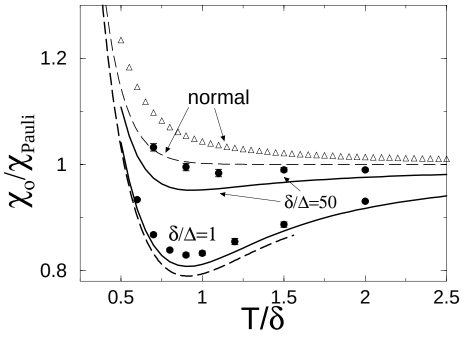

Fig. 1. Spin susceptibility for dots with odd N. Lines: ordered spectrum, $\epsilon_\alpha = \delta\alpha$. Symbols: GOE distributed ϵ_α.

ultrasmall grains. The specific heat turns out to be a very sensitive probe to detect pairing correlations; in even and odd number of electron case the results for the leading contributions ($T, \tilde{\lambda}_\Omega \ll \delta$) are respectively

$$c_{Ve}(T) = \frac{3\pi^2\tilde{\lambda}_2^3}{\delta^2 T} \mathrm{e}^{-\frac{2\tilde{\lambda}_2}{T}} \ , \quad c_{Vo}(T) = \frac{3}{2}\pi^2\zeta(3)\frac{T^2}{\delta^2} \ .$$

$c_{Ve}(T)$ shows a gap depending on the pairing interaction only, while for grains with odd N the dominant low-lying excitations are the single-electron ones and $c_{Vo}(T)$ shows the normal power-law behavior [8]. These analytical results have been recently confirmed and extended to a wider range of λ and T by numerical analysis [7]. Within the same method we get that at low temperatures the susceptibility χ of samples with even N is exponentially suppressed, so we concentrate on the odd-N susceptibility χ_o. Fig. 1 shows the result obtained by functional technique [7]. The reentrance due to the competition between parity and pairing [6] is present also when the disorder is taken into account. Our study shows that unequivocal signatures of superconducting correlations turn out to be the anomalies of the specific heat and spin-susceptibility at low and intermediate temperature.

Interplay between exchange and pairing: The Hamiltonian (1) is marked by an interplay between the pairing term which favors ground states with lower spin and the exchange term which determines the opposite trend. This interplay can be analyzed by means of the probability distribution $P_S(J/\delta)$ of finding a ground state with spin S as a function of the exchange strength J. We obtain the following distributions in the limits of weak exchange and pairing $0 < J - J_S^* \ll \lambda \ll \delta$ [9]:

$$P_S(J/\delta) = \frac{C_S}{\delta^{(S+1)(2S-1)}} [J(J-J_S^*)]^{(S+1)(2S-1)/2}, \quad (2)$$

where the C_S are dimensionless constant depending only on the spin S (e.g. $C_1 = \pi^2/3$ and $C_{3/2} = 9\pi^4/50$) and J_S^* is a cutoff value of the exchange below which the spin probability vanishes (e.g. $J_1^* = \tilde{\lambda}_2$ and $J_{3/2}^* = 2\tilde{\lambda}_3/3$). We have assumed the renormalized pairing constant $\tilde{\lambda}_{2S}$ to be fixed, apart from the mesoscopic fluctuations. The normal distributions are recovered in the limit $\lambda \to 0$ [10]. We have compared the expected behavior of $P_1(J/\delta)$ with the numerical data in the systems with an even number of electrons. The values of the exchange strength J/δ were considered where the probabilities of larger spin $S > 1$ ground states are still negligible and hence the two-level approximation is valid. The probabilities follow the expected behavior in the case $\delta \gg \Delta$. In the region with stronger pairing $\delta > \Delta$, they display in the low-probability zone a "tail", where they assume larger values than the expected ones, and then follow the expected behavior. The tail signals the failure of the approximation of the fixed two-level renormalized pairing constant and increases when the ratio δ/Δ decreases due to the fact that the fluctuations increase with the pairing. The fitted renormalized two-levels pairing constants $\tilde{\lambda}_2$ follow a universal functions of the ratio δ/Δ, given by the equation $\tilde{\lambda}_2 = \delta/\ln(a\delta/\Delta)$ [4], with the value of the low-frequency cutoff constant $a = 1.721$. In the samples with an odd number of electrons the expected behavior of the spin-3/2 probability is reproduced also. The tails are, in the limit of statistics, not present. This arises because the pairing interaction (and the fluctuations of the renormalized constant) is weakened by the presence of an unpaired electron in the spin 1/2 ground state. The fitted three-levels renormalized pairing constants $\tilde{\lambda}_3$ present the same universal behavior as in the systems with an even number of electron, apart from the value of the constant $a = 1.679$.

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