

Josephson nanocircuit in the presence of linear quantum noise

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Abstract

We derive the effective hamiltonian for a charge-Josephson qubit in a circuit with no use of phenomenological arguments, showing how energy renormalizations induced by the environment appear with no need of phenomenological counterterms. This analysis may be important for multiqubit systems and geometric quantum computation.

Key words: Josephson effect; quantum computation; decoherence

Josephson junction based nanocircuits have been proposed for the implementation of quantum gates[1] and quantum coherent behavior has been recently observed[2] in charge-based devices (charging energy E_C larger than the Josephson energy E_J). Decoherence in these devices is due to several sources[1], as fluctuations of the circuit, backaction of the measuring apparatus or noise due to background charges in the substrate[3]. Fluctuations of the circuit are modeled by coupling the system to an environment of harmonic oscillators[4] which mimics the external impedances (see Fig.1). An effective hamiltonian H_{eff} is considered, which represents a spin-boson model (or a multistate version), the central variable being the charge Q in the island and the environment being fixed in a phenomenological way[1]. The environment produces decoherence and energy shifts, which may in principle be large. In dissipative quantum mechanics shifts are usually treated either by introducing counterterms[4] or by writing H_{eff} in terms of renormalized quantities[4]. We present here a model for the electromagnetic environment and we derive a multistate H_{eff} using no phenomenological argument. This has two motivations. First in principle bare circuit param-

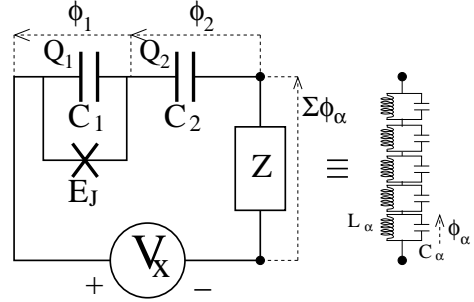


Fig. 1. The Cooper pair box in an electromagnetic environment. Fluctuations are due to the impedance $Z(\omega)$, which is modeled by a suitable infinite LC transmission line

eters are well defined and tunable, so we want to know precisely how this reflects on H . Second the role of induced shifts, which is minor in the devices of Refs.[2], may be crucial in various situations (e.g. geometric quantum computation[5], dynamics of registers and error correction devices).

We consider the Cooper pair box[1] of Fig.1. The external impedance is modeled by a suitable LC transmission line and the Lagrangian of the system is

$$L = \sum_{i=1,2} \frac{C_i \dot{\phi}_i^2}{2} - V_J(\phi_1) + \sum_{\alpha} \left[\frac{C_{\alpha} \dot{\phi}_{\alpha}^2}{2} - \frac{\phi_{\alpha}^2}{2L_{\alpha}} \right]$$

where $\dot{\phi}$ are voltage drops and the Josephson energy is

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$V_J(\phi_1) = -E_J \cos(2e\phi_1/\hbar)$. The environment is fully specified by the elements C_α and L_α . The circuit is introduced by the constraint $\dot{\phi}_1 + \dot{\phi}_2 + \sum_\alpha \dot{\phi}_\alpha = V_x$, which allows to eliminate one variable and to write

$$L = \frac{C_\Sigma \dot{\eta}^2}{2} - V_J(2e(\kappa_2\Phi - \eta)/\hbar) + L_b \quad (1)$$

$$L_b = \frac{1}{2} C_e \dot{\Phi}^2 + \sum_\alpha \left[\frac{C_\alpha}{2} \dot{\phi}_\alpha^2 - \frac{1}{2L_\alpha} \phi_\alpha^2 \right]$$

where $\eta = \kappa_2\phi_2 - \kappa_1\phi_1$, $C_\Sigma = \sum_i C_i$, $C_e = C_1 C_2 / C_\Sigma$, $\kappa_{1,2} = C_{1,2} / C_\Sigma$, and $\Phi = V_x - \sum_\alpha \dot{\phi}_\alpha$.

We next diagonalize L_b and obtain the form

$$L_b = \sum_\alpha \left[\frac{m_\alpha}{2} \dot{x}_\alpha^2 - \frac{m_\alpha \omega_\alpha^2}{2} x_\alpha^2 \right] - C_e V_x \sum_\alpha d_\alpha \dot{x}_\alpha$$

In order to determine the parameters, notice that L_b describes a series $C_e - Z$ circuit, with $\sum_\beta d_\beta \dot{x}_\beta = \sum_\alpha \dot{\phi}_\alpha = V_Z$. By comparing the linear response with the known classical dynamics of V_Z (as explained in Refs.[4]) we determine the spectral density ($\omega > 0$)

$$J'(\omega) = \sum_\alpha \frac{\pi d_\alpha^2 \delta(\omega - \omega_\alpha)}{2m_\alpha \omega_\alpha} = \text{Re} \left[\frac{Z(\omega)/\omega}{1 + i\omega Z(\omega)C_e} \right]$$

We stress that L_b is quadratic therefore the procedure above is an exact way to perform the diagonalization, which uses classical circuit theory as a tool. It's validity for quantum harmonic oscillators is guaranteed by the Ehrenfest theorem.

We get rid of the Φ in the potential term in Eq.(1), we perform a (canonical) transformation $\chi = \eta - \kappa_2\Phi$ and obtain the total Lagrangian $L = L_a + L_b$ where

$$L_a = \frac{C_\Sigma}{2} (\dot{\chi} + \kappa_2 V_x - \kappa_2 \sum_\alpha d_\alpha \dot{x}_\alpha)^2 + V_J(2e\chi/\hbar)$$

One can verify that the variable canonically conjugated to χ is the charge Q in the island. The Hamiltonian corresponding to L reads

$$H = \frac{Q^2}{2C_1} + Q \kappa_2 \sum_\alpha \frac{d_\alpha}{m_\alpha} p_\alpha - E_J \cos\left(\frac{2e}{\hbar} \chi\right) + \sum_\alpha \left[\frac{p_\alpha^2}{m_\alpha} + \frac{1}{2} \omega_\alpha^2 x_\alpha^2 \right] + C_e V_x \sum_\alpha \frac{d_\alpha}{m_\alpha} p_\alpha$$

where p_α are conjugated to x_α and we used the relation $\sum_\alpha d_\alpha^2/m_\alpha = 1/C_e$. The system variable Q is coupled with the momenta p_α of the environment.

A further canonical transformation of the environment ($\tilde{x}_\alpha = p_\alpha/(m_\alpha \omega_\alpha)$, $\tilde{p}_\alpha = -m_\alpha \omega_\alpha x_\alpha$) yields a hamiltonian where Q is coupled with the coordinates

$$H_{eff} = \frac{Q^2}{2C_1} + V_J\left(\frac{2e}{\hbar} \chi\right) + \kappa_2 Q \sum_\alpha c_\alpha \tilde{x}_\alpha + \sum_\alpha \left[\frac{\tilde{p}_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \tilde{x}_\alpha^2 \right] + C_e V_x \sum_\alpha c_\alpha \tilde{x}_\alpha$$

where $c_\alpha = d_\alpha \omega_\alpha$ and we introduce the spectral density $J(\omega) = \pi \sum_\alpha \delta(\omega - \omega_\alpha) c_\alpha^2 / (2m_\alpha \omega_\alpha) = \omega^2 J'(\omega)$.

If we isolate the dc bias $V_x(t) = V_x + \delta V_x(t)$ and redefine $\tilde{x}_\alpha + c_\alpha C_e V_x / (m_\alpha \omega_\alpha^2) \rightarrow x_\alpha$ we finally obtain

$$H_{eff} = \frac{Q^2}{2C_1} - \kappa_2 V_x Q + V_J\left(\frac{2e}{\hbar} \chi\right) + Q \kappa_2 \sum_\alpha c_\alpha x_\alpha + \sum_\alpha \left[\frac{p_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} x_\alpha^2 \right] + C_e \delta V_x(t) \sum_\alpha c_\alpha x_\alpha$$

Notice that the capacitance C_1 (and not C_Σ) enters the Q^2 term and if we put $c_\alpha = 0$ we do not obtain the Cooper pair box hamiltonian. This is correct because the environment represents global fluctuations of the circuit, not only of Z . Notice that a static Q shifts the equilibrium positions of the oscillators and also produces a Q -dependent shift of the zeroes of their energies. We can single out the corresponding term and focus on it. In the situations described in Refs.[4] this term should be canceled by introducing a counterterm, because we have informations only about renormalized effective parameters of the model. In our case we have information about the bare parameters therefore there is no reason to cancel the Q -dependent shift of the zero of the oscillator energies. It can be reabsorbed in the charging energy if we write the oscillator hamiltonian using the shifted values $x_\alpha + c_\alpha \kappa_2 Q / (m_\alpha \omega_\alpha^2)$, which produces the extra term $-\kappa_2^2 Q^2 \sum_\alpha c_\alpha^2 / (2m_\alpha \omega_\alpha^2) = -Q^2 / (2C_e)$ and

$$H_{eff} = \frac{Q^2}{2C_\Sigma} - \kappa_2 V_x Q + V_J\left(\frac{2e\chi}{\hbar}\right) + C_e \delta V_x \sum_\alpha c_\alpha x_\alpha + \sum_\alpha \left[\frac{p_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \left(x_\alpha + \frac{c_\alpha \kappa_2 Q}{m_\alpha \omega_\alpha^2} \right)^2 \right] \quad (2)$$

which reduces to the non dissipative form by letting $c_\alpha = 0$. This is a convenient starting point for a weak coupling analysis also because a static shift in the equilibrium points of the oscillators has no effect even if it is large.

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