

# Decoherence and $1/f$ noise in Josephson qubits

E. Paladino<sup>a,b,1</sup>, L. Faoro<sup>c</sup>, A. D'Arrigo<sup>a</sup> and G. Falci<sup>a</sup>

<sup>a</sup>*Dipartimento di Metodologie Fisiche e Chimiche (DMFCI), Università di Catania, Viale A. Doria 6, 95125 Catania, Italy*

<sup>b</sup>*NEST-INFM, Scuola Normale Superiore Pisa, Italy*

<sup>c</sup>*Institute for Scientific Interchange (ISI) & INFM, Viale Settimo Severo 65, 10133 Torino, Italy*

---

## Abstract

We analyse decoherence in Josephson qubits induced by background charges responsible for  $1/f$  noise. To this end we introduce a quantum mechanical model of a discrete environment. The discrete nature of the environment leads to a number of new features which are mostly pronounced for slowly moving charges.

*Key words:* Josephson devices; decoherence ; quantum computation

---

Solid state nanodevices are studied as possible implementation of a quantum computer [1] because of advantages which may come from their tunability and integrability. Recently quantum coherent behavior has been detected in superconducting nanocircuits [2]. On the other hand decoherence may represent a serious limitation [3], due to the presence of low energy excitations in the solid state environment. Sources of decoherence in Josephson nanocircuits are fluctuations of the surrounding circuit, quasiparticle tunneling, fluctuating background charges (BC) and flux noise [4,5]. We focus on decoherence due to a discrete environment which describes the most serious limitation for Josephson charge qubits, i.e. noise originated from fluctuating charged impurities, which are random traps for single electrons in dielectric materials close to the island. These fluctuations cause the  $1/f$  noise observed in Single Electron Tunneling devices [6,7]. A charge-Josephson qubit [4] in a BCs environment has been recently studied [8] by the following Hamiltonian (the same model for the BCs has been used in Ref [9])

$$H = \frac{\delta E_c}{2} \sigma_z - \frac{E_J}{2} \sigma_x - \frac{1}{2} \sigma_z \sum_i v_i b_i^\dagger b_i + \sum_i H_i$$

where the qubit is described by Pauli matrices, with couplings  $\delta E_c \equiv E_C(1 - C_x V_x/e)$ , the tunable charging

bias and  $E_J$ , the Josephson energy. Each BC is modeled by a localized electronic level ( $b_i, b_i^\dagger, \varepsilon_{ci}$ ) close to the junction which may tunnel, with tunneling amplitude  $T_{ki}$  to a band ( $c_{ki}, c_{ki}^\dagger, \varepsilon_{ki}$ )

$$H_i = \varepsilon_{ci} b_i^\dagger b_i + \sum_k [T_{ki} c_{ki}^\dagger b_i + \text{h.c.}] + \sum_k \varepsilon_{ki} c_{ki}^\dagger c_{ki}$$

The coupling is such that each BC produces a bistable extra bias  $v_i$  for the qubit. In the relaxation regime each BC has total switching rate  $\gamma_i = 2\pi N(\varepsilon_{ci})|T_i|^2$  ( $N$  is the density of states of the band,  $|T_{ki}|^2 \approx |T_i|^2$ ). We take  $\gamma_i$  distributed according to  $\sim 1/\gamma$  for  $\gamma \in [\gamma_m, \gamma_M]$ , to reproduce  $1/f$  noise [10].

We found [8] that decoherence due to each BC depends qualitatively on the ratio  $g_i = v_i/\gamma_i$ . A *weakly coupled* BC ( $g_i \ll 1$ ) behaves as a source of gaussian noise, fully characterized by the power spectrum  $s_i(\omega)$  of the fluctuations of the extra bias  $v_i b_i^\dagger b_i$ . Instead a *strongly coupled* BC ( $g_i \gg 1$ ) may show pronounced features of its discrete character, as saturation effects and dependence on initial conditions. Here we present results for different kind of operations and it turns out that decoherence is sensitive to different physical aspects of the BC environment.

We first consider decoherence *during* the time evolution of the qubit, relevant for single shot measurements. For  $\delta E_c = 0$  we use the Heisemberg equations of motion [8] and find that when strongly coupled (or slow)

---

<sup>1</sup> Corresponding author E-mail: epaladino@dmfci.unict.it

BCs are present the dephasing rate does not depend only on the total power spectrum  $S(\omega) = \sum_i s_i(\omega)$ . Indeed different sets of BCs with the same  $S(\omega)$  may determine different dephasing, as a result of saturation of the strongly coupled BCs,  $g_i \gg 1$ . A more powerful analysis can be performed for  $E_J = 0$ , since in this case we have pure dephasing and the model can be solved exactly [8]. For incoherent dynamics of the BCs we found an analytic form for the decay of the coherences in the density matrix of the qubit (in the charge basis)

$$\rho_{10}(t) \propto \exp\left[-\sum_i \Gamma_i(t) + i\delta E_i(t)\right] \quad (1)$$

A spectral analysis of the effect of a  $1/\gamma$  distribution shows that slow fluctuators  $\gamma < \langle v_i \rangle$  are ineffective (Fig.1). This is entirely due to the discrete nature of the environment and does not occur if we approximate the effect of BCs with an oscillator environment.

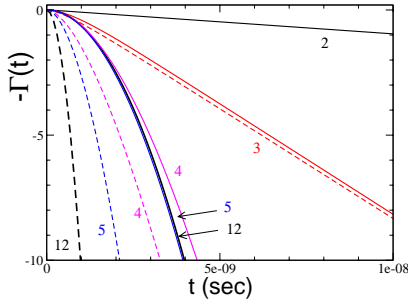


Fig. 1. Saturation of slow BCs for a  $1/f$  spectrum. The parameters chosen give typical experimental noise levels. Labels are the number of decades of noise included,  $[\gamma_m, \gamma_M = 10^{12} \text{ Hz}]$ . Dashed lines correspond to the oscillator approximation.  $\Gamma(t)$  is almost unaffected by strongly coupled charges.

We now study the effect of measurements repeated in the total time  $t_{meas}$ . Decoherence comes also from the dynamics of the BCs for  $t \in [0, t_{meas}]$  (this is analogous to inhomogeneous broadening in Free Induction Decay experiments [7,11]). The result Eq.(1) has to be averaged over different initial conditions of the BCs. This amounts to use averaged initial conditions in Eq.(1). Fast enough charges are averaged during  $t_{meas}$  so a rough approach is to use the thermal averages for charges with  $\gamma \geq \tilde{\gamma} = \min(\langle v_i \rangle, 1/t_{meas})$ . Results in Fig.2 (dotted lines) show that in this case dephasing, at short times, is approximately given by the oscillator environment approximation with a lower cutoff taken at  $\omega \sim \tilde{\gamma}$  and clarify the validity of the estimate proposed in Ref.[12]. We also checked more accurate averaging procedures (solid lines in Fig.2), which account for time correlations of the BCs dynamics, and found that they are important only if  $t_{meas} \sim \langle v_i \rangle$ .

We finally consider a “charge” echo process [7]. A semiclassical analysis yields an expression similar to

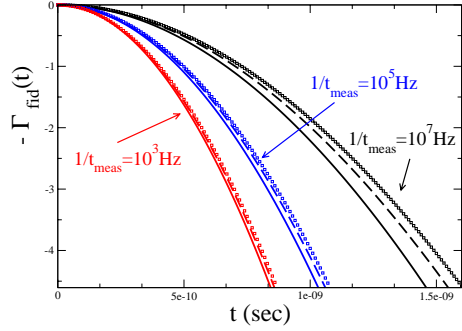


Fig. 2. Effect of different averages over initial conditions of the BCs. The dashed line correspond to the oscillator environment approximation with a lower cutoff taken at  $\omega \sim \tilde{\gamma}$ . Power spectrum as in Fig.1,  $\langle v_i \rangle \approx 10^7 \text{ Hz}$ ,  $\gamma_m = 1 \text{ Hz}$ ,  $\gamma_M = 10^9 \text{ Hz}$ .

Eq.(1), where as expected only BCs with  $\gamma \geq 1/t$  ( $t$  is half of the duration of the total echo process) play a role. If this BCs are “weakly coupled” (i.e.  $t \ll 1/\langle v_i \rangle$ ) as probably occurs in the experiment [7]) then they behave as an oscillator environment. This result clarifies the validity of the estimate proposed in Ref.[12].

## Acknowledgements

We thank R. Fazio with whom part of this work has been performed. Support from EU (IST-FET SQUBIT) and INFN-PRA-SSQI is acknowledged.

## References

- [1] Y. Makhlin *et al.* Rev. Mod. Phys. **73** (2001) 357 and references therein.
- [2] Y. Nakamura *et al.*, Nature **398** (1999) 786; D. Vion *et al.*, Science, **296** (2002) 886.
- [3] W. Zurek, Physics Today, **44** (1991) 36; M.Nielsen and I.Chuang, *Quantum Computation and Quantum Communication*, Cambridge University Press, (2000)
- [4] Y. Makhlin, G. Schön and A. Shnirman, Nature **398** (1999) 305; A. Shnirman, G. Schön and Z. Hermon, Phys. Rev. Lett. **79** (1997) 2371.
- [5] L. Tian *et al.*, Proceedings of the NATO-ASI on *Quantum Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics*, I.O. Kulik and R. Ellitioglu Eds. (Kluwer Pub. 2000), pag. 429.
- [6] A.B. Zorin *et al.*, Phys. Rev. B, **53** (1996) 13682;
- [7] Y. Nakamura *et al.* Phys. Rev. Lett. **88** (2002) 047901.
- [8] E. Paladino *et al.*, Phys. Rev. Lett. **88** (2002) 228304.
- [9] R. Bauernschmitt, Y.V. Nazarov, Phys. Rev. B, **47** (1993) 9997.
- [10] M.B. Weissman, Rev. Mod. Phys. **60** (1988) 537.
- [11] C.P. Slichter, *Principles of Magnetic Resonance and Quantum Information*, 3rd ed. (Springer-Verlag, 2000)
- [12] A. Cottet *et al.* in *Macroscopic Quantum Computing and Coherence*, D.V. Averin, B. Ruggiero, P. Silvestrini Eds. (Kluwer Pub. 2001)