

# The effect of large quantum fluctuation on the noise of a single-electron transistor

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## Abstract

We theoretically investigate the noise of a single-electron transistor at large quantum fluctuation of charge out of equilibrium. We show that the charge noise is suppressed due to the charge renormalization caused by the quantum fluctuation. However the fluctuation is not strong enough to wash out the charge quantization. We find that the renormalization effect reduces the performance of a single-electron electrometer.

*Key words:* single-electron transistor; quantum fluctuation; renormalization effect; energy sensitivity

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In a small metallic island where the charging energy  $E_C$  exceeds the temperature  $T$  (we use the unit  $k_B = 1$ ), the Coulomb interaction affects transport properties. The resulting phenomenon called the Coulomb blockade has been attracted much attention in the last decade. Especially, the quantum fluctuation of charge in a single-electron transistor (SET) is one of the basic problems in this field. The quantum fluctuation is quantitatively characterized by a parameter, dimensionless conductance  $\alpha_0 = R_K / ((2\pi)^2 R_T)$ , where  $R_K = h/e^2$  is the quantum resistance and  $R_T$  is the parallel tunneling resistance of the source and the drain junction. Recently, there has been much development in theoretical investigations in the whole range of the tunneling strength. In the *weak tunneling regime*, where the life-time broadening of a charge state level is much smaller than the typical level spacing of charge states,  $\alpha_0 < 1$ , the lowest two charge states well describe the low-energy physics. At the degeneracy point where the energy difference between two charge states  $\Delta_0 \propto E_C$  is zero, the charge number fluctuates greatly. At this point, the conductance and charging energy renormalization below the Kondo temperature  $T_K = E_C / (2\pi) e^{-1/(2\alpha_0)}$  has been predicted[1].

Though previous investigations have revealed much about the quantum fluctuation, they are limited to the discussion on average values. In order to understand quantum nature of the charge fluctuation, investigations on the statistical property of carriers, i.e. the noise, are required. The noise in the weak tunneling regime is also important for practical applications, because it determines the performance of SET electrometers [3,4].

In this paper, we discuss the charge noise and the energy sensitivity. They are calculated based on the Schwinger-Keldysh approach and the drone fermion mapping technique[2]. We reformulated and extended the resonant tunneling approximation[1] to the noise expression in a charge conserving way. In a previous paper[2], we discuss the region  $eV \gg T_K$  where the renormalization effect is unimportant. In this paper we discuss the opposite region  $eV < T_K$  and 0 K by using the expression obtained in Ref. [2].

Figure 1 (a) shows the bias voltage dependence of the normalized charge noise. As  $\alpha_0$  increases, it decreases because of the charge renormalization. The charge renormalization means that the charge noise becomes the same form as the orthodox theory with renormalized parameters, the renormalized conductance  $\tilde{\alpha}_0 = z\alpha$ , charging energy  $\tilde{\Delta}_0 = z\Delta_0$  and charge  $\tilde{e} = ze: 4\tilde{e}^2\tilde{\Gamma}_+ \tilde{\Gamma}_- / (\tilde{\Gamma}_+ + \tilde{\Gamma}_-)^3$ . The transition rate

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of a electron tunneling into (out of) the island  $\tilde{\Gamma}_{+(-)}$  is estimated by the golden rule. The renormalization factor is given by  $z = 1/(1 + 2\alpha_0 \ln(E_C/(|eV|/2)))$ . It is noticed that the result is inconsistent with a naive expectation; from the Johnson-Nyquist formula  $S_{QQ} = 4R_T T C^2$ , the charge noise is expected to be proportional to  $z^{-3}$ .

Though the normalized charge noise is suppressed, it diverges at  $eV = \Delta_0 = 0$ . It means that the charge changes by “one” at the degeneracy point even at the large quantum fluctuation. This is confirmed by the fact that the slope of excitation energy dependence of the average charge diverges at the degeneracy point (Fig. 1 (b)).

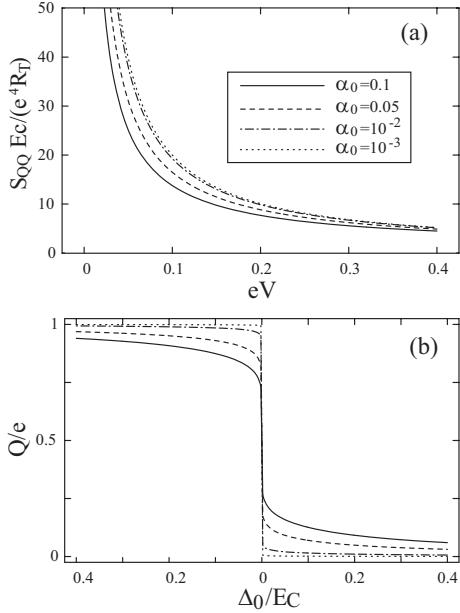


Fig. 1. (a) The bias voltage dependence of the charge noise at  $\Delta_0 = 0$  and 0 K. (b) The average charge as a function of  $\Delta_0$  at  $eV = 0$ . Solid, dashed, dot-dashed and dotted lines show results for  $\alpha_0 = 0.1, 0.05, 10^{-2}$  and  $10^{-3}$ , respectively.

We discuss the renormalization effect on the performance of a SET electrometer. Figures 2 (a) and (b) show the excitation energy dependence of the slope of  $I$ - $V$  characteristic and the energy sensitivity for various bias voltage with large  $\alpha_0$ . As the bias voltage decreases, the peak shifts rightwards because of the charging energy renormalization[1]. Moreover the peak hight is strongly suppressed. This is because both the charging energy and the conductance are renormalized, the slope  $\partial I / \partial \Delta_0$  is approximately proportional to  $z^2$ . As  $S_{II} \propto z$  and  $S_{QQ} \propto z$ , the energy sensitivity  $(\hbar/2)\sqrt{S_{QQ} S_{II}} / |\partial I / \partial \Delta_0|$  is inversely proportional to  $z$ . This fact means that the renormalization effect reduces the energy sensitivity as shown in Fig. 2 (b).

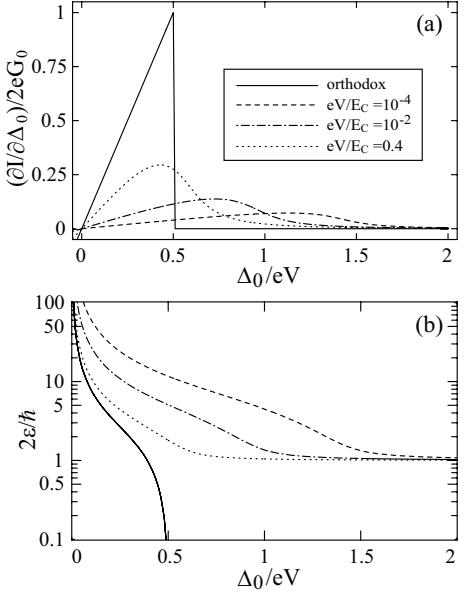


Fig. 2. The excitation energy dependence of (a) the slope of the average current and (b) the energy sensitivity for  $\alpha_0 = 0.1$  at  $eV/E_C = 10^{-4}$  (dashed line),  $10^{-2}$  (dot-dashed line) and 0.4 (dotted line) at 0 K. Solid lines shows results of the orthodox theory.  $G_0 = 1/(R_L + R_R)$  is the series conductance.

In conclusion, we theoretically investigated effect of the renormalization on the noise. We showed that the charge noise is suppressed due to the charge renormalization. However the quantum fluctuation does not wash out the charge quantization for arbitrary  $\alpha_0$ . The renormalization effect reduces the performance of SET electrometer because both the conductance and the charging energy renormalization tend to suppress the slope of  $I$ - $V$  characteristic.

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