Collective ferromagnetic states of degenerate atomic Fermi gas with two components in a trapping potential

Takaaki Sogo [a,1], Hiroyuki Yabu [a]

[a] *Department of Physics, Tokyo Metropolitan University, 1-1 Minami-Ohsawa, Hachioji, Tokyo 192-0397, Japan*

Abstract

We discuss spin asymmetry of the ground states for the trapped spin-degenerate (two components) gases of fermionic atoms with a repulsive interaction between different spin components. The asymmetric state in different spin components, "collective ferromagnetic state", is shown to appear as the ground state in the system with a strong repulsive interaction or with large particle numbers. A condition for the asymmetric ground state to occur is obtained.

Key words: gas of Alkali metal atoms ; Fermi degeneracy ; ferromagnetic state

Recent developments of laser trapping and cooling of atomic gases realized quantum degenerate phenomena for Alkali atoms with multi-components. One of interesting physics in such systems is the phase structures in the ground states, for which several studies have been done up to now.

In this paper, we discuss the trapped fermi-degenerate atomic gas with two-components: $m = \pm 1/2$ (magnetic quantum number) states of spin-1/2 atoms, or two of the hyperfine states of fermionic atoms with a larger total spin ($F > 1/2$). We denote these components as "up" and "down".

At $T = 0$, the ground state is determined by minimum condition of the energy: $E_{\rm tot} = E_{\rm kin} + E_{\rm int}$, where $E_{\rm kin,int}$ are the kinetic and interaction energies of the system. Generally, the $E_{\rm kin}$ in the asymmetric states with unequal numbers of up/down atoms is larger than that in the symmetric ones with equal numbers of up/down atoms because of the Pauli blocking. Thus, for a weakly interacting gas or for a small number of atoms with small $E_{\rm int}$, the symmetric state becomes the stable ground state. Next, we consider $E_{\rm int}$ that is more effective in strongly interacting systems. The interactions between up-up/down-down components can be neglected due to the Pauli blocking effects, so that atom pairs with the up and down components mainly contribute to the interaction energy $E_{\rm int}$. Since the asymmetric states have less up-down component pairs and the smaller $E_{\rm int}$ than the symmetric ones, they become the ground states in strongly interacting or large particle number systems.

Corresponding to the metallic magnetism, we call the asymmetric state "(collective) ferromagnetic state" and the symmetric state "paramagnetic state'. In the following, we show occurrence of the ferromagnetic state and investigate densities, energies, conditions for that state.

We consider a $T = 0$ system of two-component Fermi gas trapped in an isotropic harmonic oscillator potential; densities of spin up/down components are denoted by $\rho_\uparrow(r)$ and $\rho_\downarrow(r)$. To describe the ground-state behaviors of the system, we use the Thomas-Fermi approximation, where the total energy of the system is a functional of the densities:

$$E = \int d^3 r \Big[\sum_{\sigma=\uparrow,\downarrow} \Big\{ \frac{\hbar^2}{2m} \frac{3}{5} (6\pi^2)^{\frac{2}{3}} \rho_\sigma^{\frac{5}{3}} + \frac{1}{2} m\omega^2 r^2 \rho_\sigma \Big\} + g\rho_\uparrow\rho_\downarrow \Big]. \quad (1)$$

[1] E-mail:tsogo@comp.metro-u.ac.jp

In (1) the m and ω are the fermion mass and the oscillator frequency of the trapping potential, and g is the strength of the coupling constant given by $g = 4\pi\hbar^2 a/m$ where a is the s-wave scattering length. In the present paper, we discuss a system of repulsive interaction, so that the parameter g should be positive ($g > 0$).

we introduce the scaled dimensionless variables: $n_\sigma = 128\rho_\sigma a^3/9\pi, x = 4ar/3\pi\xi^2, \tilde{E} = 2^{18}a^8 E/3^7\pi^6\xi^8\hbar\omega$, where $\xi = \sqrt{\hbar/m\omega}$ is the oscillator length. Using these variables, Eq. (1) becomes independent of the parameters (m, ω, g):

$$\tilde{E} = \int \mathrm{d}^3x \big[\sum_{\sigma=\uparrow,\downarrow} \Big(\frac{3}{5} n_\sigma^{\frac{5}{3}} + x^2 n_\sigma \Big) + n_\uparrow n_\downarrow \big]. \qquad (2)$$

The densities $n_{\uparrow,\downarrow}$ are obtained by the stationary condition and the stability condition of the energy $\tilde{E}$ with a constraint on the scaled fermion number $\tilde{N} = \sum_\sigma \int \mathrm{d}^3x n_\sigma = 2^{13}a^6N/3^5\pi\xi^6$, i.e., $\frac{\delta}{\delta n_\sigma}(\tilde{E} - \lambda\tilde{N}) = 0$ and $\det\{\frac{\delta^2}{\delta n_\sigma \delta n_{\sigma'}}(\tilde{E} - \lambda\tilde{N})\} \geq 0$, where λ is the Lagrange multiplier for the scaled fermion number constraint. As λ is determined by $\tilde{N}$, the scaled fermion number $\tilde{N}$ is the only parameter that determines the ground-state properties of the system.

Fig. 1(a) and (b) show density profiles for $\tilde{N} = 0.2$ and for $\tilde{N} = 0.7$. In Fig. 1, the up and down atoms give identical density distributions for the scaled fermion number $\tilde{N} = 0.2$, while asymmetry shows up for $\tilde{N} = 0.7$. This suggest that the ground state become paramagnetic for small $\tilde{N}$ and ferromagnetic for large $\tilde{N}$. In the ferromagnetic state, the density distributions become symmetric in the outside region and asymmetric in the inside region. For much larger $\tilde{N}$, the density of down atoms become zero in the central region (complete asymmetry inside).

Fig. 2 shows the energy of the ground state measured from that of the paramagnetic state E_{para} against the scaled fermion number $\tilde{N}$. The critical value $\tilde{N}_{\mathrm{c}}$ for the transition from paramagnetic states to ferromagnetic states become ~ 0.53 as shown in Fig. 2.

The energy difference between ferromagnetic and paramagnetic states $|\tilde{E}_{\mathrm{ferro}} - \tilde{E}_{\mathrm{para}}| = \Delta\tilde{E}$ (for $\tilde{N} > \tilde{N}_{\mathrm{c}}$) is very small in comparison with the total energy $\tilde{E}$. However, compared with an one-particle excitation energy at the Fermi surface, which can be estimated from the scaled chemical potential $\tilde{\mu}$ defined by $\tilde{\mu}N = \lambda\tilde{N}$ with $N = \sum_\sigma \int \mathrm{d}^3r\rho_\sigma$, the ratio $\Delta\tilde{E}/\tilde{\mu}$ becomes 10^{13} in case of ^{40}K with the harmonic oscillator frequency $\omega = 1000\,\mathrm{Hz}$. Thus, we should say that the energy difference between ferromagnetic and paramagnetic states are fairly large.

We discussed the possibility of transition to ferromagnetic states in the two-component fermi gas using the Thomas-Fermi approximation. The ferromagnetic ground states become stable when $\tilde{N} > \tilde{N}_{\mathrm{c}} = 0.53$. Taking ^{40}K atoms (mass $m = 0.649 \times 10^{-25}$ kg) trapped in a harmonic oscillator potential with $\omega = 1000\,\mathrm{Hz}$, the ferromagnetic state occurs for $N > 10^{13}$ for the scattering length $a = 169a_0$ (a_0: Bohr radius). If the scattering length can be increased by the Feshbach resonance for ^{40}K, as observed experimentally, the ferromagnetic ground states can be obtained for a smaller fermion number: simple estimation gives, $a = 1200a_0$ in $N > 10^8$. The use of heavier elements, e.g. Sr or Yb, is also effective for the ferromagnetic states. We expect that the combination of these methods may lead to the experimental achievements of the ferromagnetic states of the trapped fermionic gas in the future.

More detailed argument and application of the present report are given in Ref. [1].

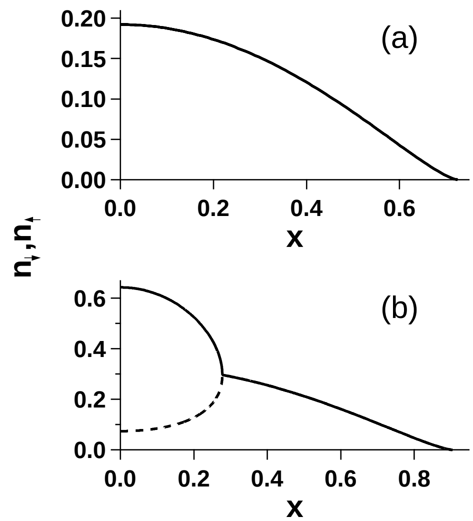

Fig. 1. Density profiles for (a) $\tilde{N} = 0.2$ and (b) $\tilde{N} = 0.7$. Solid line and dashed line are $n_\uparrow$ and $n_\downarrow$.

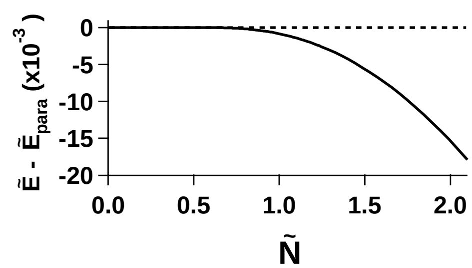

Fig. 2. Scaled energy of the ground state $\tilde{E}$ measured from that of paramagnetic state $\tilde{E}_{\mathrm{para}}$ against scaled particle number $\tilde{N}$.

References

[1] T. Sogo, H. Yabu, cond-mat/0205638 and references therein.