

# Inhomogeneous d-wave state and lattice distortions in the three-band Hubbard model of high- $T_c$ cuprates

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## Abstract

Striped superconductivity (SC) with lattice distortions is investigated based on the three-band Hubbard model of high- $T_c$  cuprates. A stable inhomogeneous striped state is determined in the low-temperature tetragonal (LTT) phase using a quantum variational Monte Carlo method. The ground state has vertical or horizontal hole-rich arrays coexisting with incommensurate magnetism and SC. The SC order parameter oscillates according to the inhomogeneity in the antiferromagnetic background with its maximum in the hole-rich region.

*Key words:* stripes, incommensurate SDW, lattice distortion, LTT phase, d-wave pairing

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*Introduction* Over the last decade the oxide high- $T_c$  superconductors have been investigated intensively. The mechanism of superconductivity (SC) has been extensively studied using various two-dimensional (2D) models of electronic interactions. Studies of these models over the last decade indicated that the  $d$ -wave SC is induced from the electronic repulsive interaction[1–7]; significantly it has been established that the SC condensation energy and the magnitude of order parameter are in reasonable agreement with the experimental results in the optimally doped case.[8,9]

In the underdoped region many physical properties remain unresolved. A striped state has been proposed based on incommensurate SDW and CDW correlations observed in the neutron-scattering measurements.[10,11] The linear doping dependence of incommensurability in the underdoped region supports a striped structure and suggests a relationship between magnetism and SC.[11] A relationship between the SDW, CDW orders and a crystal structure has also been suggested; in the LTT phase, the CDW order is stabilized, while no well defined incommensu-

rate CDW peaks were observed for the orthorhombic systems.[12]

*Lattice distortions and superconductivity* The Hamiltonian is given by the three-band Hubbard model for  $d$  and  $p$  orbitals with the modulated transfer terms:

$$\begin{aligned}
 H = & H_{pd}^0 + t_{pp} \sum_{i\sigma} v_i [p_{i+\hat{y}/2,\sigma}^\dagger p_{i+\hat{x}/2,\sigma} - p_{i+\hat{y}/2,\sigma}^\dagger p_{i-\hat{x}/2,\sigma} \\
 & - p_{i-\hat{y}/2,\sigma}^\dagger p_{i+\hat{x}/2,\sigma} + p_{i-\hat{y}/2,\sigma}^\dagger p_{i-\hat{x}/2,\sigma} + h.c.] \\
 & + t_{pd} \sum_{i\sigma} [u_{i\hat{x}} d_{i\sigma}^\dagger p_{i+\hat{x}/2,\sigma} - u_{i,-\hat{x}} d_{i\sigma}^\dagger p_{i-\hat{x}/2,\sigma} \\
 & + u_{i\hat{y}} d_{i\sigma}^\dagger p_{i+\hat{y}/2,\sigma} - u_{i,-\hat{y}} d_{i\sigma}^\dagger p_{i-\hat{y}/2,\sigma} + h.c.] \\
 & + \frac{K_{pd}}{2} \sum_i (u_{i\hat{x}}^2 + u_{i,-\hat{x}}^2 + u_{i\hat{y}}^2 + u_{i,-\hat{y}}^2) + \frac{K_{pp}}{2} \sum_i 4v_i^2 \\
 & + U_d \sum_i d_{i\uparrow}^\dagger d_{i\uparrow} d_{i\downarrow}^\dagger d_{i\downarrow}.
 \end{aligned} \tag{1}$$

$H_{pd}^0$  is the kinetic term, and  $\hat{x}$  and  $\hat{y}$  represent unit vectors in the  $x$ - and  $y$ -direction, respectively.

The wave function with the inhomogeneous spin structure is made from solutions of the Hartree-Fock Hamiltonian with the potential  $\sum_{i\sigma} [\delta n_{di} - \sigma(-1)^{x_i+y_i} m_i] d_{i\sigma}^\dagger d_{i\sigma}$  where  $\delta n_{di}$  and  $m_i$  are expressed

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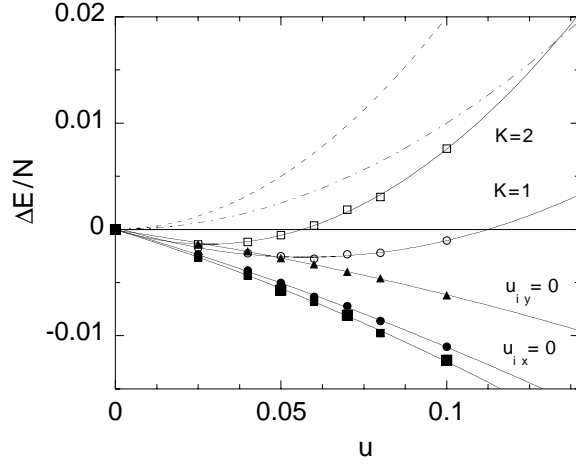


Fig. 1.  $\Delta E = E(u=0) - E(u)$  per site as a function of transfer deformation in  $t_{pd}$  units. The parameters are  $t_{pp} = 0.4$  and  $U_d = 8$ . The hole-rich stripes are in the  $y$ -direction. The energy gains for (A)  $u_{i\hat{x}} = 0$  (solid circles), (B)  $u_{i\hat{y}} = 0$ , (triangles) and (C) (solid squares) are shown. The elastic energy  $Ku^2/2$  is shown by the dashed line ( $K = 1$ ) and dash-dotted line ( $K = 2$ ). The open symbols display the summations of  $\Delta E$  and the elastic energy corresponding to solid symbols.  $K = 1$  is assigned for (A) and  $K = 2$  for (C).

by modulation vectors  $Q_s$  and  $Q_c$  representing the spin and charge part, respectively.[9,13,14] The wave function is constructed from the solution of Bogoliubov-de Gennes equation for the SC order parameters in the  $d$ -electron part:  $\Delta_{i,i+\hat{x}} = \Delta_s \cos(Q_x(x_i + \hat{x}/2))$ ,  $\Delta_{i,i+\hat{y}} = -\Delta_s \cos(Q_x x_i)$ , where  $Q_x = \pi/4$  at  $1/8$ -filling. The SC order parameter oscillates so that the amplitude has a maximum in the hole-rich region and is suppressed in the hole-poor region. The wave function is taken to be Gutzwiller.[15,16]

In the low-temperature tetragonal (LTT) phase, there is a tilting axis on which the copper and oxygen atoms never move.[17] When the tilting axis is in the  $\mu$ -direction, the deformation of  $t_{pd}$  in the  $\mu$ -direction vanishes:  $u_{i\hat{\mu}} = 0$ . We consider the following cases assuming that the stripes are parallel to the  $y$ -axis:

- (A)  $u_{i\hat{x}} = 0, u_{i\hat{y}} = u \cos(2Q_x x_i), v_i = u \cos(2Q_x x_i)$ ,
- (B)  $u_{i\hat{x}} = u \cos(2Q_x x_i), u_{i\hat{y}} = 0, v_i = u \cos(2Q_x x_i)$ ,
- (C)  $u_{i\hat{x}} = u_{i\hat{y}} = u \cos(2Q_x x_i), v_i = u \cos(2Q_x x_i)$ ,

where  $Q_x = 2\pi\delta$ ,  $u$  is the amplitude of deformation of  $t_{pd}$  and  $t_{pp}$  and  $u_{i\hat{\mu}} = u_{i,-\hat{\mu}}$ . The energy gain per site defined as  $\Delta E/N = (E(u=0) - E(u))/N$  is presented in Fig.1 as a function of  $u$  in  $t_{pd}$  units. According to Harrison's rule,  $t_{pd}$  is expected to vary as  $d^{-n}$  with  $n \approx 7/2$ ,  $d$  being the Cu-O bondlength. The elastic energy is estimated as  $E_{el} = \frac{1}{2} C d^3 (\frac{\delta d}{d})^2 = \frac{1}{2} C d^3 \frac{1}{n^2} u^2 \equiv \frac{K}{2} u^2$ . The constant  $C$  is estimated as  $C \approx 1.7 \times 10^{12}$  dyne/cm<sup>2</sup> = 1.7 eV/Å<sup>3</sup>, [18] and then  $K$  is of the order

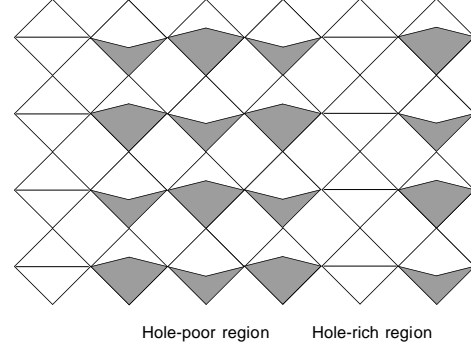


Fig. 2. Schematic structure of lattice distortions and stripes obtained by the present evaluations where the hole-rich arrays are perpendicular to the tilting axis. The shaded square represents distorted CuO unit cell.

of 1eV:  $K \approx 1\text{eV}$  since  $d \approx 2\text{\AA}$ . As shown in Fig.1 the energy is lowered when the stripes are perpendicular to the tilting axis. Although the state in the case (C) has a lowest energy for fixed  $u$ , its total energy may be higher than that for (A) since the elastic energy will increase due to complex lattice distortions. We show schematically stable striped state in Fig.2.

*Summary* In this paper we have investigated the inhomogeneous ground state with the lattice distortions based on the three-band model. The stable striped state has hole-rich arrays being perpendicular to the tilting axis of the lattice distortions in the LTT phase.

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