

# Quantum transport in disordered normal metal/ unconventional superconductor junctions

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## Abstract

We calculate tunnel conductance for disordered normal metal/ unconventional superconductor junctions where superconducting order parameter has broken time reversal symmetry (BTRS), i.e.,  $d_{x^2-y^2} + i s$ . It is shown that reflectionless tunneling phenomena appears since zero-energy resonant states caused by the internal phase of  $d_{x^2-y^2}$ -pair potential is broken by BTRS states. It is also shown that splitting of zero bias conductance peak due to BTRS in conductance spectra remains whereas the height of the peak is suppressed by the randomness.

*Key words:* N/S junction; ZBCP; disorder

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The promising symmetry in the superconducting state of high  $T_C$  cuprates is considered to be  $d_{x^2-y^2}$ -wave. Reflecting zero-energy resonant states formed at the surface of  $d$ -wave superconductor caused by the internal phase of pair potential, conductance peak (ZBCP) appears at zero bias in tunnel spectra of normal metal/  $d$ -wave superconductor junctions [1,2]. Recent studies revealed that the order parameter has broken time-reversal symmetry (BTRS) in the form of  $d_{x^2-y^2} + i s$ -wave, and that the ZBCP is split [3–5]. The purpose of this work is to clarify the effect of disorder on the electrical transport in the junction consisting of normal metal and  $d$ -wave superconductor with BTRS.

We consider a normal metal (N)/ insulator(I)/ superconductor (S) junctions as shown in inset of Fig. 1. The junction is stacked along (100) direction of a square lattice with lattice spacing  $a$  which is unit of length hereafter. A single-orbital tight-binding model is used to describe the system. The BCS mean field Hamiltonian  $\mathcal{H} \equiv H + \Delta$  is given by

$$H = -t \sum_{\langle \mathbf{i}, \mathbf{j} \rangle, \sigma} c_{\mathbf{i}\sigma}^\dagger c_{\mathbf{j}\sigma} + \sum_{\mathbf{i}, \sigma} v_{\mathbf{i}} c_{\mathbf{i}\sigma}^\dagger c_{\mathbf{i}\sigma} - \mu \hat{n}, \quad (1)$$

$$\Delta = \sum_{\mathbf{i}, \mathbf{j} \in S, \sigma} (\Delta_{\mathbf{ij}} c_{\mathbf{i}\sigma} c_{\mathbf{j}-\sigma} + \text{H.c.}), \quad (2)$$

where  $\mathbf{i}$  and  $\mathbf{j}$  are sites,  $\sigma$  electron spin,  $t$  hopping integral which is unit of energy hereafter,  $v_{\mathbf{i}}$  potential at site  $\mathbf{i}$ ,  $\mu$  chemical potential,  $c_{\mathbf{i}\sigma}^{(\dagger)}$  annihilation (creation) operator, and  $\hat{n}$  number operator. As for the order parameter of S, we divide  $\Delta_{\mathbf{ij}}$  into two parts as  $\Delta_{\mathbf{ij}} = \Delta_{\mathbf{ij}}^d \cos \alpha + i \Delta_{\mathbf{ij}}^s \sin \alpha$  where  $\Delta_{\mathbf{ij}}^d$  and  $\Delta_{\mathbf{ij}}^s$  have  $d_{x^2-y^2}$  and  $s$ -wave symmetry, respectively, and the parameter  $\alpha$  describe the degree of BTRS. Although BTRS states appears in S near junction interface [5], we introduce  $i \Delta_{\mathbf{ij}}^s$  in whole region of S for simplicity. We assume second neighbor pairs for  $\Delta_{\mathbf{ij}}^d$  and on-site pairs for  $\Delta_{\mathbf{ij}}^s$ . We introduce randomness in the normal lead (see inset of Fig. 1). The randomness at the disordered region is treated by random site potential  $v_{\mathbf{i}}$ . We assume that  $v_{\mathbf{i}}$  takes a random values uniformly distributed within  $\pm w/2$  at disordered region. By using the Kubo formula and the recursive Green's function method [6], we

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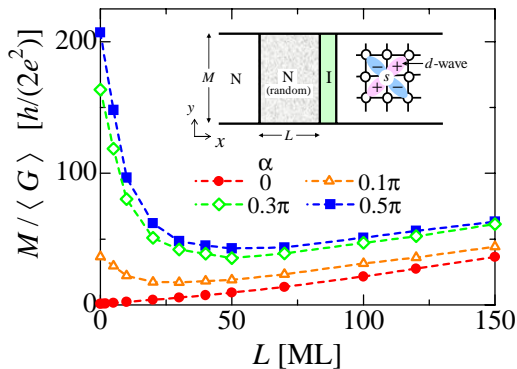


Fig. 1. Resistance as a function of the length of disordered region in normal metal. Junctions with  $\alpha = 0$  and  $\pi/2$  correspond to N/I/ $d$ - and  $s$ -wave S junctions, respectively. Inset: Schematic figure of a disordered N/I/S junction.

calculate conductance numerically for various samples which have different configuration of randomness. Average conductance  $\langle G \rangle$  is then, obtained by ensemble averaging over the samples.

We set  $M = 32$  in unit of  $a$ ,  $\Delta_0 = 0.001$ ,  $\mu = -1.0$ , and  $w = 2.0$  in unit of  $t$ . The thickness and  $v_i$  of I are chosen as 1 and 5.0, respectively, so that the transmission probability of the interface is 0.11. We also use 500 samples to carry out ensemble averaging throughout calculations. The mean free path and the localization length of the disordered region are roughly estimated as 6.5 and 140, respectively.

Figure 1 shows calculated results of the junction resistance at zero bias as a function of the length  $L$  of disordered region in normal metal. In N/I/ $s$ -wave S junction ( $\alpha = 0.5\pi$ ), the resistance has a minimum at a certain value of  $L$ , which phenomena is known as “reflectionless tunneling” [7]. In contrast to that, the resistance in N/I/ $d$ -wave S junction ( $\alpha = 0$ ) shows monotonic increase with  $L$ . The reason is that strong resonance occurs at the interface since the sign of  $d$ -wave pair potentials that incident and reflected quasiparticles feel is opposite, then, the interface resistance is suppressed [8]. As  $\alpha$  increases from zero and the  $s$ -wave component of the pair potential increases relative to the  $d$ -wave one, the resistance of the junction increases and tends to the result of N/I/ $s$ -wave S junction.

Calculated results of conductance spectra of the junction with BTRS ( $\alpha = 0.01\pi$ ) are shown in Fig. 2 for several values of  $L$ . Here, the conductance of N/I/S junction is normalized by that of N/I/N junction. It can be seen that ZBCP is split due to the  $s$ -component of the pair potential, that is, BTRS breaks the zero-energy resonant states at the interface. As the length  $L$  of the disordered region in normal metal increase, the height of the ZBCP decreases, however, the split of ZBCP still remains.

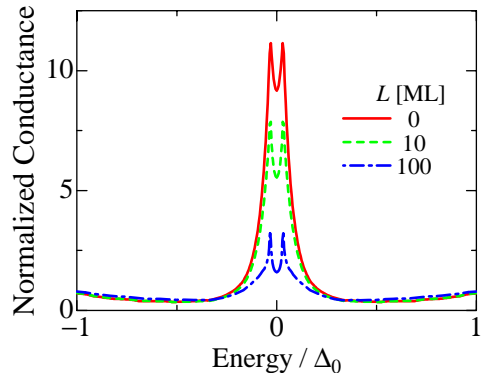


Fig. 2. Conductance spectra calculated for N/I/S junctions with  $\alpha = \pi/100$ . Conductance is normalized by that of N/I/N junction where pair potential  $\Delta_0$  of S is set zero.

In summary, we have calculated the electrical conductance for disordered normal metal/ unconventional superconductor junctions with BTRS, that is,  $d_{x^2-y^2} + i s$ . It has been shown that reflectionless tunneling phenomena appears since zero-energy resonant states at the junction interface caused by the internal phase of  $d_{x^2-y^2}$ -pair potential is broken by BTRS and, then, the interface resistance is increased. It has been also found that splitting of ZBCP due to BTRS in conductance spectra remains whereas the height of the peak decreases as the length of the disordered region in normal metal increases. In this work, we did not determine the pair potential self-consistently. However, the pair potential, especially its  $s$ -wave component, near the junction interface could be affected by the randomness. Therefore, self-consistent calculation in disordered junction is desired.

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