Simulations of dynamics for wavepackets made by two-component Bose-Einstein condensates in Alkali-atom gases

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Abstract

In relation with remarkable time-evolution measurements of JILA reported in D.S.Hall et al, Phys. Rev. Lett. **81** (1998) 1539,where Bose condensates of ^{87}Rb for different hyperfine states Ψ_1=|F=1,m=-1⟩ and Ψ_2=|F=2,m=1 ⟩ can be confined under various conditions in harmonic traps, we calculate the dynamic simulation for wavepackets Ψ_1 and Ψ_2 by the coupled time-dependent Gross-Pitaevskii equations. In fact, we give relative sag to both states in center of the trap following to experimental conditions: As a result of calculation, Ψ_1 and Ψ_2 show the phase-separation indicating the vibrational motions with the out-of-phase behavior. As for variations of centers of masses and interpenetrative motions of wavepackets with bouncing back, the agreements between the present numerical results and experiments of JILA are quite well.

Key words: dynamics of wavepackes; The coupled Gross-Pitaevskii equations; vibrations of centers of masses; phase-separations

1. Introduction

Recently, JILA-group [1] has reported the noteworthy experiments, in which Bose condensates of ^{87}Rb for different hyperfine states Ψ_1=|F=1,m=-1⟩ and Ψ_2=|F=2,m=1 ⟩ can be confined under various artificial conditions in harmonic traps. To explain these experimental results, we reproduce the dynamics of wavepackets by computer simulation under the corresponding conditions adopted in their observation.

2. Theoretical model

We treat the following coupled time-dependent Gross-Pitaevskii equations [2] for two condensates:

$$i\frac{\partial\Psi_j}{\partial t} = -\frac{1}{2}\nabla^2\Psi_j + \alpha_{jj}|\Psi_j|^2\Psi_j + \alpha_{jk}|\Psi_k|^2\Psi_j$$

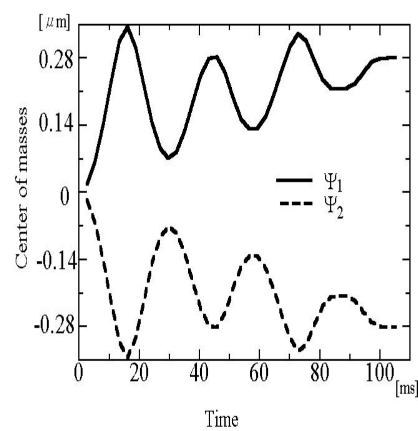

Fig. 1. The relative motions of centers of masses of two condensates. Here, the relative sag is 3% [1] of the width of the distribution prior to expansion. We can find the out-of-phase behavior in the vibration.

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$$+\frac{1}{2}[\rho^2 + (z + (-1)^j z_0)^2]\Psi_j,$$

where (j, k) =(1,2) and (2,1). The coupling constants α_{jk} is expressed by scattering lengths a_{ij} and the number of atoms N as $4\pi N a_{jk}$.(The units of length and energy are those of the harmonic oscillator.) In calculation of these non-linear equations for wavepackets in traps, we investigate the behavior of Ψ_1 and Ψ_2 concentrating our attention on the following case: The relative sag made by $\pm z_0$ is given to both states in center of the trap. As a result of calculation by the use of the same values of parameters and conditions as those in experiments [1,3], Ψ_1 and Ψ_2 show the phase-separation indicating the vibrational behavior. It should be noted that no parameter is used so as to fit the calculation to results of experiments.

3. Calculated results and discussion

The relative motions of centers of mass of two condensates is shown in Fig.1, where the relative sag is 3% [1] of the width of the distribution prior to expansion. Here, we can find the out-of-phase behavior in the vibration. In detail, the distributions at 64.8ms are shown in Fig.2, where the phase separation is found in spite of interpenetrative features. Further, time evolution of wavepackets are also illustrated in Fig.3. We would like to emphasize that the vibrations with the out-of-phase behavior appear in Ψ_1 and Ψ_2 showing interpenetrative motions without the distinctive boundary. With proceeding time, the amplitude of the vibration decreases and phase separation becomes stable. In the region of $0 \leq t \leq 40ms$, these characteristic features of bouncing back in calculation agree quite well with those in experiments [1]. As for $t \geq 40ms$, the effect of energy relaxation becomes essential in experiments. In this region, the adjustable parameter related with relaxation time should be taken into account to the calculation. If we adopt the empirical method in which energy relaxation is considered as the term for the imaginary time, we can obtain the well agreement with experiments even in $t \geq 40ms$. However, the mechanism of the energy relaxation and its value are the open problems.

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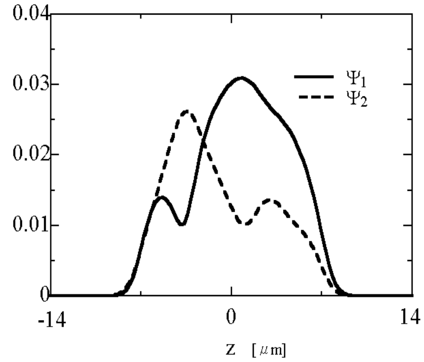

Fig. 2. The distributions at 64.8ms.

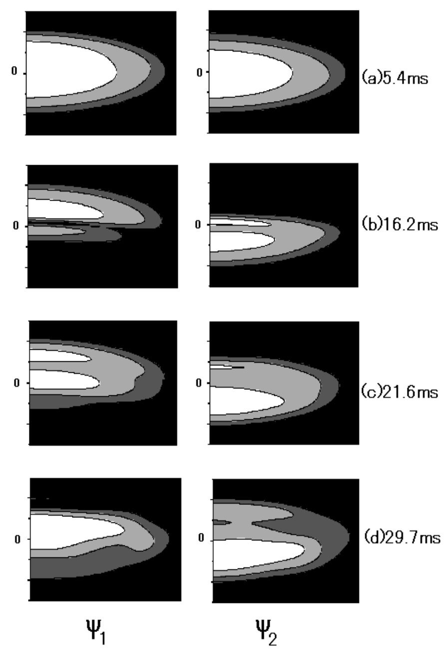

Fig. 3. The evolution of wavepackets. Here, the adopted parameters and conditions are same as Fig.1.

References

[1] D.S.Hall, M.R.Matthews , J.R.Ensher , C.E.Wieman , and E.A.Cornell ,Phys. Rev. Lett. **81** (1998) 1539.

[2] K.Doi and Y.Natsume, J.Phys.Soc.Jpn **70** (2001) 167.

[3] In fact, the ratios of intraspecies and interspecies scattering lengths a_{11}, a_{22} and a_{12} are given as $a_{11} : a_{22} : a_{12} = 1.03 : 0.97 : 1.00$. (The value of average of these three lengths is 55(3)Å.)