

# ac current due to photon-assisted tunneling in driven mesoscopic systems

Kousuke Yakubo<sup>a,1</sup>, Jun-ichiro Ohe<sup>b</sup>,

<sup>a</sup>*Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan*

<sup>b</sup>*Department of Physics, Sophia University, Kioi-cho 7-1 Chiyoda-ku, Tokyo 102-8554, Japan*

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## Abstract

Photon-assisted ac transport through coherently driven mesoscopic systems has been studied by employing a transfer-matrix method. For a driven double-barrier system as a simple model of a quantum dot, we show that the ac current due to quantum interference between different sideband states depends on wavenumbers of electrons and exhibits sideband peaks. We also suggest that such ac currents can be observed in a driven mesoscopic ring which provides a novel type of photon-assisted tunneling.

*Key words:* photon-assisted tunneling; ac transport; transfer-matrix method

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Applying a time-varying potential with frequency  $\omega$  to an electron in a resonant tunneling system, the electron can exchange energy with the oscillating potential and tunnel through additional resonant levels with keeping its coherence. This phenomenon, so-called photon-assisted tunneling (PAT), has been observed in a number of resonant systems[1]. Although ac currents are excited in such systems, most of efforts have been concentrated on dc transport in previous works on PAT.

We have theoretically studied, in this paper, ac currents in driven mesoscopic systems by employing a transfer-matrix method which enables us to compute quantitatively ac currents without consuming a large amount of computing time and memory space. Systems treated here is described by the one-dimensional Schrödinger equation,  $i\hbar\dot{\psi} = [-\hbar^2/2m^*\Delta + V_{dc}(x) + V_{ac}(x)\cos\omega t]\psi$ , where  $V_{dc}(x)$  and  $V_{ac}(x)$  are arbitrary functions of  $x$ . The transmitted wave (far from scatterers) is given by

$$\psi(x) = \sum_{n=-\infty}^{\infty} A_n e^{ik_n x} e^{-i(E+n\hbar\omega)t/\hbar}, \quad (1)$$

where  $k_n = \sqrt{2m^*(E - V_{dc} + n\hbar\omega)}/\hbar$  is the wavenumber of the  $n$ th sideband state and  $V_{dc} = V_{dc}(x \rightarrow \infty)$ . The coefficient  $A_n$  for each sideband state is calculated by the transfer-matrix method for driven mesoscopic systems[2]. From the wavefunction Eq. (1), the current density is expressed as[3]

$$j = \sum_n j_{dc}(n) + \sum_n j_{ac}(n\omega), \quad (2)$$

where

$$j_{dc}(n) = \frac{e\hbar k_n}{m^*} |A_n|^2, \quad (3)$$

and

$$j_{ac}(n\omega) = ie \sum_m k_m [A_{m+n} A_m^* e^{i(k_{m+n}-k_m)x} e^{in\omega t} + A_{m-n} A_m^* e^{i(k_{m-n}-k_m)x} e^{-in\omega t}]. \quad (4)$$

Equation (2) shows that there exist ac currents  $j_{ac}(n\omega)$  due to quantum interference between different sideband states in addition to dc current  $j_{dc}$ . Note that  $j_{ac}(n\omega)$  depends also on the wavenumber. This implies that the amplitude of the ac current is a function of the measuring point of  $j_{ac}$ .

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<sup>1</sup> E-mail: yakubo@eng.hokudai.ac.jp

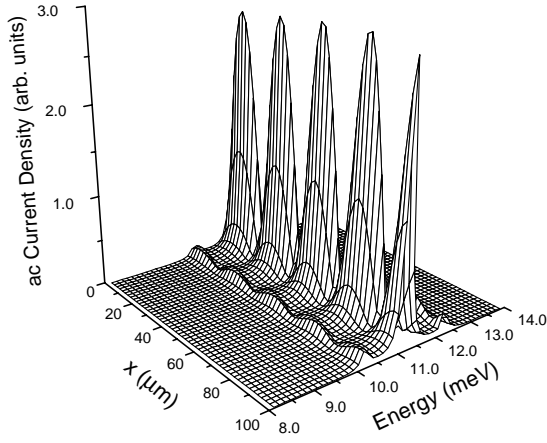


Fig. 1. ac current in a driven double-barrier system as a function of the incident electron energy and the distance from the center of  $V_{dc}$  to the measuring position.

We first demonstrate the ac current through a driven double-barrier system. The static potential constructing the double barrier is given by

$$V_{dc} = \begin{cases} V_0(x) & \text{for } x_0 - \xi/2 \leq |x| \leq x_0 + \xi/2, \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

and the time-varying potential is

$$V_{ac}(x) = \begin{cases} V_1 & \text{for } |x| \leq x_0 - \xi/2, \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

Figure 1 represents the amplitude of the ac current  $j_{ac}(\omega)$  as a function of the incident electron energy and the distance from the center of two barriers. Parameters for this calculation are  $V_0 = 30$  meV,  $V_1 = 0.1$  meV,  $\xi = 10$  nm,  $x_0 = 40$  nm, and  $\hbar\omega = 1.0$  meV. One can find a sharp resonance peak at the resonant energy level ( $\varepsilon_0 = 11.2$  meV) of this system and distinct sideband peaks at  $E = \varepsilon_0 \pm \hbar\omega$ , which provides a clear evidence of photon-assisted ac currents. As we expected,  $j_{ac}(\omega)$  is an oscillating function of the measuring position.

Due to incoherence of electrodes and the finite wavenumber character of  $j_{ac}(\omega)$ , one cannot measure directly the ac current by attaching external electrodes. One possible way is a measurement of a magnetic flux induced by  $j_{ac}(\omega)$  in a driven mesoscopic ring. In contrast to conventional PAT, quite acute and cusp-like PAT signals appear in the conductance of this system[4]. The time-varying potential given by  $V_{ac}(x) = V_1 e^{-x^2/\xi^2}$  is applied at the center of the one arm of the ring. The calculated ac current shows cusplike signals as in the case of the dc current. The ac current produces a time-varying magnetic field  $B(t)$ . The induced magnetic field is given by

$$B(t) = \frac{\pi\mu_0}{4L^2} \oint J(s,t) ds, \quad (7)$$

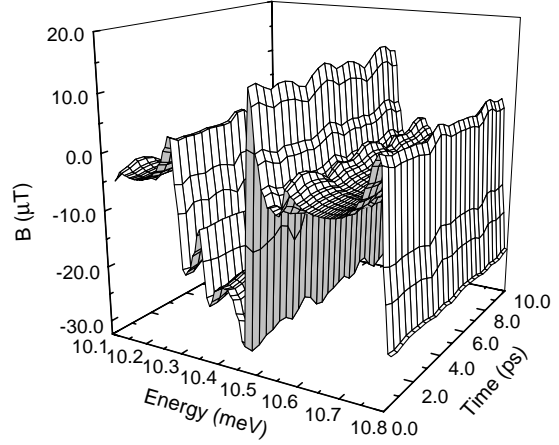


Fig. 2. Magnetic field induced by the current in the driven mesoscopic ring as a function of the incident electron energy and time.

where  $J(s,t)$  is the current within the interval  $[s, s+ds]$  on the ring at time  $t$  and  $L$  is the length of the arms. Figure 2 shows the induced magnetic field as a function of the incident electron energy and time. Here we set  $L = 1$  μm,  $\xi = 20$  nm,  $V_1 = 0.15$  meV, and  $\hbar\omega = 0.1$  meV. The induced magnetic field fluctuates in time, which results from the ac current in the ring. The time-averaged magnetic field  $\bar{B}(E)$  coming from the dc component of  $J(s,t)$  reflects the cusp-like structure of the total current (the main resonant energy  $\varepsilon_0$  is 10.37 meV). The time dependence of  $B(t)$  at a fixed energy is not a simple sinusoidal function. This is because  $J(s,t)$  includes higher harmonics. The amplitude of the fluctuating magnetic field is of the order of μT that is measurable magnitude in actual experiments.

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## References

- [1] P. K. Tien, J. P. Gordon, Phys. Rev. **129** (1963) 647.
- [2] K. Yakubo, Phys. Rev. E **57** (1998) 3602.
- [3] J. Ohe, K. Yakubo, Jpn. J. Appl. Phys. **40** (2001) 1982.
- [4] K. Yakubo, J. Ohe, J. Phys. Soc. Jpn. **69** (2000) 2170.