

# Anisotropy of upper critical field for high temperature superconductors

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## Abstract

The anisotropy of high  $T_c$  superconductors arises both from the coupling constant and the Fermi surface in the Ginzburg-Landau framework. The anisotropy of the former is expressed by  $d_{x^2-y^2}$  symmetry. The shapes of the Fermi surface of high  $T_c$  superconductors are various and depend on materials and doping ratio. In this paper, we study the anisotropy of the upper critical field  $H_{c2}$  in the  $ab$  plane with  $d_{x^2-y^2}$  symmetry and with typical shapes of the Fermi surface.

*Key words:* anisotropic upper critical field; anisotropic Fermi surface; pairing symmetry

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## 1. Introduction

The upper critical field  $H_{c2}$  of some high temperature superconductors shows interesting behavior with fourfold symmetry, when the magnetic field is applied in the  $ab$  plane. For example, the angular dependence of  $\text{Ca}_{0.5}\text{La}_{1.25}\text{Ba}_{1.25}\text{Cu}_3\text{O}_x$  [1] is different by  $\pi/4$  from that of  $\text{La}_{1.86}\text{Sr}_{0.14}\text{CuO}_4$  [2] and  $\text{Pb}_2\text{Sr}_2\text{Ca}_{0.38}\text{Cu}_3\text{O}_8$  [3].

To explain these results, we studied both the effects of anisotropic coupling constant of the pairing state and anisotropic Fermi surface, assuming a simple Fermi surface [4]. In this paper, we present the results of  $H_{c2}$ , using more realistic Fermi surface.

## 2. Formulation

The detailed derivation of the fourfold symmetry for the upper critical field in the  $ab$  plane is given in [4] and expressed near the transition temperature  $T_c$  as

$$\Delta h \equiv \frac{\Delta H_{c2}}{H_{c2}} = \frac{B_4 \cos 4\theta}{A_2} \delta, \quad (1)$$

where  $\delta = \ln(T/T_c)$ ,  $\theta$  is the angle between the magnetic field and the principal crystal axes in the  $ab$  plane and

$$A_{2n} = \frac{2(-1)^n}{(2\pi k_B T)^{2n}} \left( 1 - \frac{1}{2^{2n+1}} \right) \zeta(2n+1). \quad (2)$$

Here,  $\zeta(z)$  is the Riemann's zeta function.

The quantity which expresses the magnitude of anisotropic part is given by

$$B_4 = \frac{1}{64p_{0X}^2 \langle V_x^2 \rangle} - \frac{3A_4 \langle V_x^4 - 3V_x^2 V_y^2 \rangle}{16A_2 \langle V_x^2 \rangle^2}. \quad (3)$$

To obtain the explicit values of  $B_4$ , we assume a following expression for the energy dispersion:

$$\varepsilon(\mathbf{p}) = \sum_n C_n \varepsilon_n(\mathbf{p}_\perp) + \frac{p_z^2}{2m} \equiv \varepsilon_{ab}(\mathbf{p}_\perp) + \varepsilon_z(\mathbf{p}_z), \quad (4)$$

where

$$\varepsilon_1(\mathbf{p}_\perp) = 2 - \cos ap_x - \cos ap_y, \quad (5a)$$

$$\varepsilon_2(\mathbf{p}_\perp) = 1 - \cos ap_x \cos ap_y \quad (5b)$$

and

$$\varepsilon_3(\mathbf{p}_\perp) = 2 - \cos 2ap_x - \cos 2ap_y, \quad (5c)$$

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where  $a$  is the lattice constant in the  $ab$  plane. These are first three functions with square symmetry in the momentum space.

The velocity  $\mathbf{V}$  in (3) is obtained by

$$\mathbf{V} = \frac{\partial \varepsilon(\mathbf{p})}{\partial \mathbf{p}}. \quad (6)$$

These quantities  $\mathbf{p}$  and  $\mathbf{V}$  are defined with respect to the crystalline axes.

The function which expresses the anisotropy of the pairing state enters in the average  $\langle \dots \rangle$  at the Fermi surface as

$$\langle B \rangle = \frac{1}{N(0)} \int d\Omega(\mathbf{p}_F) \phi^2(\hat{\mathbf{p}}) N(\mathbf{p}_F) B(\mathbf{p}_F), \quad (7a)$$

$$N(0) = \int d\Omega(\mathbf{p}_F) \phi^2(\hat{\mathbf{p}}) N(\mathbf{p}_F), \quad (7b)$$

where we consider the  $d_{x^2-y^2}$  pairing state:

$$\phi(\hat{\mathbf{p}}) = \sqrt{2} \cos 2\psi. \quad (8)$$

### 3. Numerical Results and Conclusions

For numerical calculation of the equation (1) for  $\Delta h$ , we need an explicit expression of the energy dispersion and we use a set of parameters  $C_n = (0.05, 0.01, 0.015)$  eV in (4). In Fig. 1, we show the equilateral energy surface in the  $ab$  plane. These shapes resemble the results of the doping dependence of the Fermi surface, taken from the ARPES spectra for  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  [5].

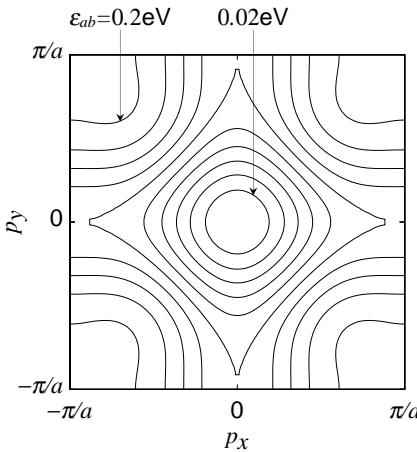


Fig. 1. Equilateral energy surfaces for the energy dispersion (4) whose parameter values are given in the text. The energy difference between two adjacent curves is  $\Delta\varepsilon = 0.02$  eV.

In Fig. 2, we present the Fermi energy  $\varepsilon_{ab}$  dependence of  $\Delta h$  for the temperature  $T = 35, 36$  and  $37\text{K}$  for  $T_{c2} = 37.5\text{K}$ . The divergence of  $\Delta h$  at  $\varepsilon_{ab} = 0.12$  eV is due to that of the density of states at  $\mathbf{p}_\perp = (\pi/a, 0)$  and its equivalent points.

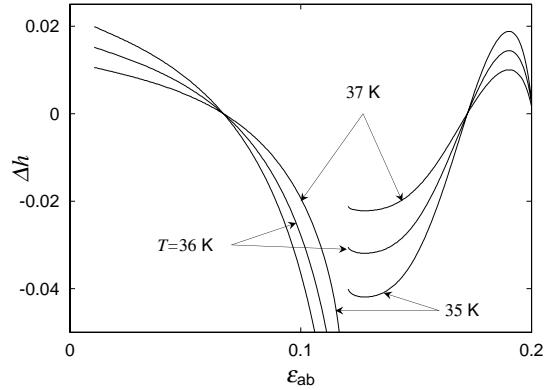


Fig. 2. The relative magnitude of anisotropy  $\Delta h$  as a function of  $\varepsilon_F$  for  $T = 35, 36$  and  $37\text{K}$  with  $T_c = 37.5\text{K}$  with the same parameter set of  $\Delta h$  Fig. 1.

The sign change of  $\Delta h$  depending on  $\varepsilon_{ab}$  corresponds to the reverse of the maximum-minimum position of the upper critical field with respect to the angle  $\theta$  found in the observations. The magnitude of  $\Delta h$  is, however, small and doping dependence is not clear in this study. To solve these problems, it is necessary to have informations of  $C_n$  on doping dependence, which is obtained from microscopic theory.

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