Vortex lattice structures in tetragonal BCS superconductors due to Fermi surface anisotropy

Anton Knigavko [1] and Frank Marsiglio

Department of Physics, University of Alberta, Edmonton, T6G 2J1, Canada.

Abstract

We investigate vortex lattice structures of tetragonal strongly type II BCS superconductors in the clean limit for the case $H||c$. Anisotropy of the Fermi surface is assumed small and treated as a perturbation. Location of the line in the $T-B$ plane separating square and rhombic vortex lattices is found.

Key words: quasiclassical theory of superconductivity, vortex lattice, anisotropy, nonlocality

Observation of a structural phase transition (SPT) in the vortex lattice (VL) of borocarbide superconductors has revived analysis of the role of anisotropies in the mixed state. If an external magnetic field, applied along [001] direction in these tetragonal crystals, increases, then a rhombic VL deforms until it transforms continuously into a square VL. The location of this SPT has been experimentally mapped very precisely in the low temperature region and well explained by the nonlocal London model[1]. Recently, neutron experiments closer to $T_{\rm c}$ also successfully determined this transition[2]. They demonstrated a tendency that the SPT line bends upwards to higher magnetic fields before it reaches the $H_{\rm c2}(T)$ line and seems never to cross it. Gurevich and Kogan proposed an explanation based on the inclusion of fluctuations into the nonlocal London model[3]. This approach emphasizes purely magnetic interaction between vortices. In this paper we present an analytical analysis of the so called core-core intervortex interactions. Their importance grows as one approaches denser vortex lattices, and may be dominant closer to the $H_{\rm c2}(T)$ line. We neglect the effects of supercurrent, which is permissible for strongly type II superconductors. An alternative approach, largely numerical, unifying both type of interactions has been proposed in a recent electronic preprint [4].

[1] Corresponding author. E-mail: aknigavko@phys.ualberta.ca

Within the framework of the quasiclassical theory of superconductivity we use the expansion in the "distance" from the $H_{\rm c2}(T)$ line[5] to obtain the location of the STP line valid far in the mixed state. In this paper we restrict ourselves to isotropic superconducting pairing, but allow for anisotropy of the Fermi surface (FS). We assume this anisotropy to be a small parameter.

We consider two dimensional FS for simplicity and model the Fermi velocity as $v = v_0\,(1+\eta\cos 4\varphi)$ with η small. The angle φ is measured relative to the [100] crystal axis. In what follows we refer to dimensionless temperature $t = T/T_{\rm c}$ and magnetic induction $b = B/(T_{\rm c}/e^* v_0^2)$. The free energy density difference between the normal and the mixed states can be written as $F = -a_{\rm h}^2/\,(2\beta_E) + \mathcal{O}(a_{\rm h}^3)$ where the Eilenberger energy parameter $\beta_{\rm E}\,(t, b; \rho, \sigma, \gamma)$ reads:

$$\beta_{\rm E} = \frac{\pi t \sum_{\omega_{\rm n}>0} \int_{FS} {\rm Re} \int_x \Delta_{\rm L} \left[\hat{P}\Delta_{\rm L}\right] \left[\hat{P}'\Delta_{\rm L}^*\right]^2}{\left[\int_x |\Delta_{\rm L}|^2\right]^2}. \qquad (1)$$

The parameter $a_{\rm h}$ is the relevant eigenvalue of the $H_{\rm c2}$ problem, while $\Delta_{\rm L}$ is the corresponding eigenfunction. The $H_{\rm c2}(t)$ line is given by $a_{\rm h}(t,b) = 0$. The $\hat{P}$ operator formally solves the Eilenberger equation $\left(\omega_{\rm n} + \frac{1}{2}\boldsymbol{v}\cdot\hat{\boldsymbol{D}}\right) f = \Delta\, g$, $\hat{\boldsymbol{D}} = \boldsymbol{\partial} - {\rm i}\boldsymbol{A}$ for the anomalous quasiclassical Green function f. Allowing for an arbitrary orientation of the vortex lattice, we work in the Landau gauge with the vector potential $\boldsymbol{A}$ rotated

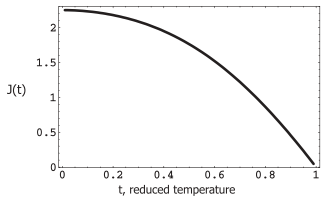

Fig. 1. The function $J(t)$ that gives temperature dependence of the anisotropic part of the eigenfunction of the H_{c2} problem.

through the angle γ about the [010] crystal axis. For $\omega_{\rm n} > 0$ the $\hat{P}$ operator reads

$$\hat{P}(\boldsymbol{v}) \equiv \int_0^\infty \mathrm{d}\tau \, \exp\left[-\tau\left(\omega_{\rm n} + \tfrac{1}{2}\boldsymbol{v}_{\rm cr}\cdot\boldsymbol{R}(\gamma)\cdot\hat{\boldsymbol{D}}_{\rm VL}\right)\right] \quad (2)$$

where the rotation operator $\boldsymbol{R}$ connects crystal coordinates and vortex lattice ones. Note that $\hat{P}' \equiv \hat{P}^*(-\boldsymbol{v})$.

To study the SPT line it is sufficient to expand everything in Eq. (1) to the first order in η. The FS averaging now becomes $\int_{FS} ... \approx \int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi}(1-\eta\cos 4\varphi) ...$ Then, the Fermi velocity entering the $\hat{P}$ operator should be expanded in η. Finally, $\Delta_{\rm L}$ contains the η corrections as well. This is clear from a look on the linearized gap equation: $\Delta_{\rm L}\ln\frac{1}{t} = 2\pi t\sum_{\omega_{\rm n}>0}\left[\frac{1}{\omega_{\rm n}} - \int_{FS}\hat{P}(\boldsymbol{v})\right]\Delta_{\rm L}$, which is also solved perturbatively in η.

In the calculation we make extensive use of the basis of lattice–shaped Landau levels ψ_m. Tetragonal symmetry of the crystal dictates $\Delta_{\rm L} = \psi_0 + \eta e^{4i\gamma}J(t)\psi_4 + \mathcal{O}(\eta^2)$, up to a normalization factor. In Fig. 1 we plot the function $J(t) \equiv J(t, h_{c2}^{(\rm iso)}(t))$ where $h_{c2}^{(\rm iso)}(t)$ is given by the isotropic H_{c2} problem. Note that anisotropic corrections to the eigenfunction grows strongly with decrease of temperature. We would like to emphasize that this temperature dependence is the main factor determining the location of the STP line on the $t-b$ plane. We present $\beta_{\rm E}$ in Eq. (1) as a function of three parameters defining the lattice shape (ρ,σ) and orientation (γ). Numerical minimization is used to find their values at the equilibrium. In this paper we concentrate on general rhombic VL only, which restricts ρ to 0 or 1/2. We find that for $\eta > 0$ the symmetry axes of rhombic VL are given by [100] and [010] crystal axis, while for $\eta < 0$ they are given by [110] and [$1\bar{1}0$]. These directions do not change with temperature.

Of crucial importance is that $\beta_{\rm E}$ depends on t and b mainly through the combination $t/\sqrt{b}$. Additional common factor $1/b$ does not influence the minimization. Therefore, for a given η it is sufficient to find the point of SPT on the $H_{c2}(t)$ line. We then draw the SPT line from this point along a parabola $b = c(\eta)\,t^2$. Shown in Fig. 2 are examples with $\eta = .015$ (the crossing at $t = .72$) and $\eta = .03$ (the crossing at $t = .86$).

As FS anisotropy increases the SPT lines moves to lower inductions b, occupying larger portion of superconducting region of $t-b$ plane. This is to be expected on the physical grounds. To see why this happens formally we write Eq. (1), after the γ minimization, as $\beta_{\rm E}(t,\sigma) = \beta_{\rm iso}(t,\sigma) - \eta\,|\beta_{\rm ani}(t,\sigma)|$ and follow the $H_{c2}(t)$ line starting from $t = 1$. For triangular lattice ($\sigma = \sqrt{3}/2$) the anisotropic correction $\beta_{\rm ani}$ stays zero at all t, while for square lattice ($\sigma = 1/2$) it grows as t decreases (see Fig. 3). Thus, square lattice ultimately wins. The larger is η the higher is temperature of the SPT. It is only for very small η that the crossing of the STP and the $H_{c2}(t)$ lines disappears at $t = 0$. This result is in contradiction with [4].

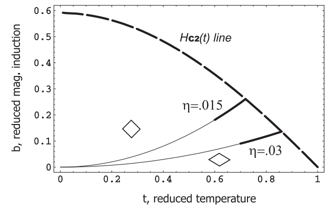

Fig. 2. Thick solid lines show the lines of the square–rhomb structural phase transition in vortex lattice with FS anisotropy indicated.

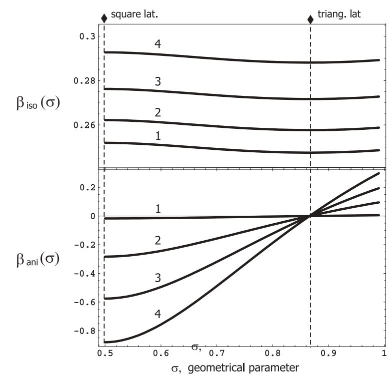

Fig. 3. The dependence of isotropic (upper panel) and anisotropic (lower panel) terms of $\beta_{\rm E}$ from Eq. (1) on geometrical parameter σ for several reduced temperatures: (1) .99, (2) .84, (3) .69, (4) .54

Within the approximations we employed we could not find "repelling" of the SPT line from the $H_{c2}(t)$ line reported in [2]. It is possible that this effect cannot be seen perturbatively in FS anisotropy.

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