Superconductivity with nonzero Chern numbers

Shuichi Murakami [a,1] Naoto Nagaosa [a,b]

[a] *Department of Applied Physics, University of Tokyo, Tokyo 113-8656, Japan*
[b] *Correlated Electron Research Center, AIST, Tsukuba Central 4, Tsukuba 305-8562, Japan*

Abstract

In strongly correlated electron systems it often happens that electronic bands acquire nonzero Chern numbers, either by spin chirality or by spin-orbit coupling. It implies that the phase of the wavefunctions cannot be defined uniquely all over the Brillouin zone, and one should be careful in defining order parameters. In this paper, we will develop a theory of condensation happening in bands with nonzero Chern numbers. We take the model on the honeycomb lattice proposed by Haldane as an example, and explain what will happen when superconductivity occurs in a band with a nonzero Chern number. It can also be related to quantization of spin Hall conductivity.

Key words: Chern number; quantum Hall effect; condensation; superconductivity

The discoveries of the integer and fractional quantum Hall effect (QHE) have thrown new light on the importance of topology in condensed materials. Electronic states can have nontrivial topological structure, and the most dramatic stage for this nontrivial topology is a two-dimensional electronic system. There the Hall conductivity can be quantized to an integer multiple of e^2/h to an enormous precision. This important role of topology is recently reemphasized in the theories of anomalous Hall effect (AHE) in ferromagnets. In contrast with the QHE, where nontrivial topology arises due to external magnetic field, nontrivial topology in the AHE arises spontaneously, i.e. without magnetic field, from either spin chirality or spin-orbit coupling. Because the spin-orbit coupling exists in every solid, all ferromagnets can be candidates for this nontrivial topology. However, to the authors' knowledge, very few works have been done to clarify this nontrivial topology in ferromagnets. One reason is that this nontrivial topology is barely visible to experiments. The Hall conductivity is the almost only chance known to see it, but it is no longer quantized when the material is metallic. Thus it is still a challenging task to observe this nontrivial topology in ferromagnets.

[1] E-mail: murakami@appi.t.u-tokyo.ac.jp

On the other hand, this nontrivial topology has a remarkable feature. If the band has nontrivial topology, i.e. nonzero Chern number [1,2], the phase of the wavefunction can no longer be a single continuous function of $\mathbf{k}$ in the whole Brillouin zone (BZ). Let us write down the Hamiltonian in the form

$$H = \sum_{\mathbf{k},i,j} c^{\dagger}_{\mathbf{k},j} H_{ji}(\mathbf{k}) c_{\mathbf{k},i}. \quad (1)$$

This Hamiltonian is diagonalized by a unitary matrix $U(\mathbf{k})$ as

$$U(\mathbf{k})^{\dagger} H(\mathbf{k}) U(\mathbf{k}) = \mathrm{diag}(E_1(\mathbf{k}), \cdots, E_n(\mathbf{k})). \quad (2)$$

We can define field operators $a_{i\mathbf{k}}$ for each eigenstate as

$$a_{i\mathbf{k}} = \sum_j U^{\dagger}(\mathbf{k})_{ji} c_{j\mathbf{k}}. \quad (3)$$

Because the matrix $U(\mathbf{k})$ is just a collection of eigenvectors of $H(\mathbf{k})$, a nonzero Chern number implies that $U(\mathbf{k})$ cannot be defined as continuous and smooth for the entire BZ. Instead the BZ should be divided into some regions V_m, in each of which $U(\mathbf{k})$ is continous. This affects a definition of field operators $a_{i\mathbf{k}}$, where i is a band index. As a result, $a_{i\mathbf{k}}$ cannot be defined as a continous operator in the whole BZ. This is an important aspect, since many theories on condensed materials are based on the assumption that $a_{i\mathbf{k}}$ is well-defined for all the wavevectors $\mathbf{k}$. The most interesting case to be focused here is the superconductivity (SC). The BCS Hamiltonian of the SC has the term $\Delta_{s_1 s_2}(\mathbf{k}) a^\dagger_{i\mathbf{k}s_1} a^\dagger_{i-\mathbf{k}s_2}$, where s_j represents the spin. When the band acquires a nonzero Chern number, this BCS term should be modified. We should then write $a_{i\mathbf{k}ms}$ for $\mathbf{k} \in V_m$, instead of $a_{i\mathbf{k}}$. Therefore, the BCS term should be written as $\Delta_{s_1 s_2}(\mathbf{k})_{m_1 m_2} a^\dagger_{i\mathbf{k}m_1 s_1} a^\dagger_{i-\mathbf{k}m_2 s_2}$, where $\mathbf{k} \in V_{m_1}$ and $-\mathbf{k} \in V_{m_2}$. Thus we see that the gap function $\Delta_{s_1 s_2}(\mathbf{k})_{m_1 m_2}$ is not a continuous (but piecewise continuous) function of $\mathbf{k}$ in the whole BZ. This is rather apart from our knowledge, and is outside existing classifications of gap functions by point-group symmetries [3,4]. It would be interesting if this new class of gap functions can be found in real materials.

By working with the topological structure of $\Delta(\mathbf{k})$ in the whole BZ, we can predict that it should have at least one zero in the whole BZ. This holds both in the singlet SC and in the triplet SC. This can be best illustrated by working on the SC on the honeycomb-lattice model proposed by Haldane [5]. This model is the simplest model where the bands have nonzero Chern numbers without external magnetic field. Note that this result asserts an existence of zeros in the whole BZ, not on the Fermi surface (FS). If this node, which generally forms a line in the three-dimensional case, crosses the FS, a number of experimental methods like specific heat, nuclear magnetic resonance, can detect it. Furthermore, if the gap opens everywhere on the FS, the spin Hall conductivity [6] is quantized. Detailed calculations and results will be presented elsewhere.

Acknowledgements

We acknowledge support by Grant-in-Aids from the Ministry of Education, Culture, Sports, Science and Technology.

References

[1] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49** 405 (1982).

[2] M. Kohmoto, Ann. Phys. (NY) **160** 343 (1985).

[3] G. E. Volovik and L. P. Gor'kov, Sov. Phys. JETP **61** 843 (1985).

[4] M. Sigrist and K. Ueda, Rev. Mod. Phys. **63** 239 (1991).

[5] F. D. M. Haldane, Phys. Rev. Lett. **61** 2015 (1988).

[6] T. Senthil, J. B. Marston, M. P. A. Fisher, Phys. Rev. **B60** 4245 (1999).