

# Effects of spin-orbit scattering on Josephson current between s-wave superconductor and $\text{Sr}_2\text{RuO}_4$

Y. Asano <sup>a</sup>, T. Hirai <sup>b</sup>, Y. Tanaka <sup>b,1</sup>, J. Inoue <sup>b</sup>, M. Sigrist <sup>c</sup>, S. Kashiwaya <sup>d</sup>

<sup>a</sup>*Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan*

<sup>b</sup>*Department of Applied Physics, Nagoya University, Nagoya 464-8063, Japan*

<sup>c</sup>*Theoretische Physik ETH-Hönggerberg CH-8093 Zürich, Switzerland*

<sup>d</sup>*National Institute of Advanced Industrial Science and Technology, Tsukuba, 305-8568, Japan*

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## Abstract

The Josephson effect in *s*-wave superconductor / insulator/  $\text{Sr}_2\text{RuO}_4$  (S/I/SRO) junctions is studied. We focus on influences of spin-orbit scattering at the junction interface on the phase-current relation ship in the Josephson current. In the presence of the spin-orbit scattering, we find that the Josephson current has a term which is proportional to  $\cos \varphi$ , where  $\varphi$  is the phase-difference between S and SRO.

*Key words:* p-wave; Josephson current; Spin-orbit scattering

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## 1. Introduction

The anisotropic superconductivity has been an important topics in condensed matter physics since unconventional superconductivity was discovered in heavy-fermion materials [1]. Anisotropic superconductivity was found in the layered perovskite  $\text{Sr}_2\text{RuO}_4$  (SRO) in a recent study [2]. Stimulated by the suggested triplet pairing symmetry, several studies on transport properties have been performed [3–7]. The Josephson effect between different superconductors of opposite parity, i.e. Josephson current between *s*-wave superconductors and SRO, have been studied both theoretically and experimentally [8–14]. In general, there is no first order Josephson coupling between spin-singlet and spin-triplet superconductors when the spin-flip transmission is absent [1,15–17]. In the experiment [8], however, Josephson current was observed when the *c* axis of SRO is parallel to the junction interface. This is because the potential step near the insulators become a source of the spin-orbit cou-

pling [15]. The Josephson current between two SRO's was also theoretically studied [18,19].

In general the Josephson current can be decomposed into a series of

$$J = \sum_{n=1}^{\infty} (a_n \cos n\varphi + b_n \sin n\varphi), \quad (1)$$

where  $\varphi$  is the phase-difference between two superconductors. In a presence of the time-reversal symmetry, in general, the Josephson current becomes an odd function of  $\varphi$ , i.e.,  $a_n = 0$ . In general SIS junctions,  $J \propto \sin \varphi$  because  $b_n$ ,  $n \geq 2$  are much smaller than  $b_1$ . However, in S/I/SRO junctions, it is known that  $b_1$  vanishes because the wavefunction of the Cooper pairs in two superconductors are orthogonal to each other. Thus the Josephson current is proportional to  $\sin 2\varphi$  in the absence of the spin-orbit scattering. In the presence of the spin-orbit scattering, a coefficient  $a_1$  or  $b_1$  may remain finite [1,15–17]. In this paper, we show that  $a_1$  becomes finite in S/I/SRO junctions.

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<sup>1</sup> E-mail: ytanaka@nuap.nagoya-u.ac.jp

## 2. Effect of spin-orbit scattering

We consider S/I/SRO junctions in two-dimensional space which corresponds to the  $ab$ -plane in SRO. The pair potential in SRO is given by  $\hat{\Delta}_p = i\mathbf{d} \cdot \boldsymbol{\sigma} \sigma_2$  with  $\mathbf{d} = \Delta_p e^{i\gamma} \mathbf{z}$ , where  $\sigma_j$  are the Pauli matrices,  $\mathbf{z}$  is a unit vector in the  $c$  axis of SRO and  $e^{i\gamma} = \bar{k}_x + i\bar{k}_y$ . The wavenumber in  $x$  and  $y$  direction on the Fermi surface are  $\bar{k}_x = k_x/k_F$  and  $\bar{k}_y = k_y/k_F$ . The current flows in the  $x$  direction and the junction is at  $x = 0$ , where we consider the potential barrier  $V_0\delta(x)$ . The spin-orbit scattering is expressed by the Hamiltonian,

$$H_{so} = \frac{-i\hbar^2}{(2mc)^2} \boldsymbol{\sigma} \cdot (\nabla V(r) \times \nabla). \quad (2)$$

In this study, we express spin-orbit scattering at the junction by the interface term  $-iV_s/k_F\delta(x)\partial_y\sigma_3$  since the space-derivative of the barrier-potential only has a  $x$  component in the two-dimensional system. The Josephson current is calculated from [17]

$$J = \frac{e}{2} T \sum_{\omega_n} \mathbf{I}, \quad (3)$$

$$\mathbf{I} = \frac{N_c}{2} \int_{-\pi/2}^{\pi/2} d\gamma \text{Tr} [\hat{\Delta}_0 \hat{a}_1 - \hat{\Delta}_0^\dagger \hat{a}_2], \quad (4)$$

where  $\hat{\Delta}_0 = \Delta_0 i\sigma_2$  is the pair potential in a  $s$ -wave superconductor,  $\hat{a}_1$  and  $\hat{a}_2$  are the Andreev reflection coefficients from the electron (hole) branch to the hole (electron) branch,  $N_c = Wk_F/\pi$  is the number of propagating channels on the Fermi surface, and  $W$  is the width of the junction. In the limit of high potential barrier ( $z_0 = mV_0/k_F \gg 1$ ), we obtain

$$\mathbf{I} = \frac{-2N_c\Delta_p \cos\varphi}{z_0^4\Omega_0} \int_{-\pi/2}^{\pi/2} d\gamma \cos^2\gamma \times \frac{z_0 z_s \Omega_0 \Omega_p \sin^2\gamma + \Delta_0 \Delta_p \cos^2\gamma \sin\varphi}{\omega_n^2 + \Delta_p^2 \sin^2\gamma}, \quad (5)$$

where  $\Delta_0(\Delta_p)$  is the amplitude of the pair potential in S (SRO),  $\Omega_{0(p)} = \sqrt{\omega_n^2 + \Delta_{0(p)}^2}$ , and  $z_s = mV_s/k_F$  represents a strength of the spin-orbit scattering. In the absence of the spin-orbit scattering, the Josephson current is not proportional to  $\sin\varphi$  but proportional to  $\sin 2\varphi$  as shown in the second term in Eq. (5). This is because the wavefunction of the Cooper pair in spin-singlet superconductors and that in spin-triplet superconductors are orthogonal to each other. In the presence of the spin-orbit scattering at the junction interface, a term proportional to  $\cos\varphi$  appears in the Josephson current as shown in the first term of Eq. (5). This apparent shift of the Josephson phase  $\varphi$  by  $\pi/2$  appears due to the fact that the Josephson coupling mediated by spin-orbit scattering involves the transverse p-wave pairing component (here  $k_y$ ) only which has a phase of  $\pi/2$  in our definition. Thus the stable

junction is reached at  $\varphi_0 = \pi/2$ . When  $\gamma = 0$ , we note that the denominator of Eq. (5) goes to zero in the limit of the zero temperature. This is because that the zero-energy resonance states [20–23] are formed at the interface for quasiparticles with perpendicular injection to the interface. It is expected that the temperature dependence of the Josephson current is seriously influenced by this state [18,19,21–23]. In the forthcoming paper, we would like to clarify this in detail.

## References

- [1] M. Sigrist and K. Ueda: Rev. Mod. Phys. **63** (1991) 239.
- [2] Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J. G. Bednorz and F. Lichtenberg: Nature **372** (1994) 532.
- [3] M. Yamashiro, Y. Tanaka and S. Kashiwaya: Phys. Rev. B **56** (1997) 7847.
- [4] C. Honerkamp and M. Sigrist: J. Low. Temp. Phys. **111** (1998) 895.
- [5] M. Yamashiro, Y. Tanaka and S. Kashiwaya: J. Phys. Soc. Jpn. **67** (1998) 3364.
- [6] N. Yoshida, Y. Tanaka, J. Inoue and S. Kashiwaya, J. Phys. Soc. Jpn. **68** (1999) 1071.
- [7] T. Hirai, N. Yoshida, Y. Tanaka, J. Inoue and S. Kashiwaya, J. Phys. Soc. Jpn. **70** (2001) 1885.
- [8] R. Jin, Y. Liu, Z. Q. Mao and Y. Maeno: Europhys. Lett. **51** (2000) 341.
- [9] A. Sumiyama, T. Endo, Y. Oda, Y. Yoshida, A. Mukai, A. Ono and Y. Onuki: Physica C **367** (2002) 129.
- [10] M. Yamashiro, Y. Tanaka and S. Kashiwaya: J. Phys. Soc. Jpn. **67** (1998) 3364.
- [11] Y. Hasegawa, K. Machida and M. Ozaki: J. Phys. Soc. Jpn. **69** (2000) 336.
- [12] Y. Tanaka and S. Kashiwaya, J. Phys. Soc. Jpn. **68** (1999) 3485.
- [13] Y. Tanaka and S. Kashiwaya, J. Phys. Soc. Jpn. **69** (2000) 1152.
- [14] C. Honerkamp and M. Sigrist: Prog. Theor. Phys. **100** (1998) 53.
- [15] V.B. Geshkenbein and A.I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 306 (1986) [JETP Lett. **43**, 395 (1986)].
- [16] A. Millis, D. Rainer, and J. A. Sauls, Phys. Rev. B **38**, 4504 (1988).
- [17] Y. Asano: Phys. Rev. B **64** (2001) 224515.
- [18] Y. S. Barash, H. Burkhardt and D. Rainer: Phys. Rev. Lett. **77** (1996) 4070.
- [19] Y. Asano: J. Phys. Soc. Jpn. **71** (2002) to be published.
- [20] Y. Tanaka and S. Kashiwaya: Phys. Rev. Lett. **74** (1995) 3451.
- [21] Y. Tanaka and S. Kashiwaya: Phys. Rev. B **53** (1996) 11957.
- [22] Y. Tanaka and S. Kashiwaya: Phys. Rev. B **56** (1997) 892.
- [23] S. Kashiwaya and Y. Tanaka: Rep. Prog. Phys. **63** (2000) 1641.