RPA Study of Collective Excitations in the Bose-Fermi Mixed Atomic Gases with Large Excess of Bosons

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Abstract

We study collective excitations in the bose-fermi mixed condensate of atomic gases with large excess of bosons based on the self-consistent Hartree-Fock(HF) and Random Phase Approximation (RPA). We investigate response of the system to a monopole type external field, and find that the fermionic strength is concentrated in a single collective state for a repulsive boson-fermion interaction. The dynamical structure factor for this state shows that fermions are correlated through the bose condensate.

Key words: Bose-Einstein condensate ; Fermi Degeneracy ; gases of Alkali-metal atoms ; collective oscillation

Recent experiments suggest that the quantum degeneracy regime has been achieved in the bose-fermi mixed gas of alkali-metal atoms [1]. The system consists of particles which obey different quantum statistics, and would be an interesting playground to study the role of statistics in the dynamical properties of many-body systems.

In the present report we study collective oscillations in the bose-fermi mixed system with large excess of bosons over fermions using the self-consistent HF plus RPA method. We consider, in particular, the monopole response of fermions and study the role of bose condensate which mediates among fermions. In the mixed system with a similar number of bosons and fermions, the latter have a much broader distribution than bosons because of the Pauli principle, and thus the boson-fermion interaction plays only a minor role in the dynamical properties of fermions [2]. In contrast, for the system with large excess of bosons, the fermions are confined within the bose condensate having a large density overlap. This could give rise to a stronger effect of the boson-fermion interaction, and eventually, to the effective fermion-fermion interaction mediated by the bose condensate. Below we briefly describe the theoretical framework and then present numerical results for the strength distribution and the dynamical structure factor of a collective oscillation.

We consider a spin-polarized bose-fermi mixed gas of atoms trapped in a spherical harmonic oscillator potential at $T = 0$. The Hamiltonian of this system is given by

$$\hat{H} = \hat{H}_0 + \hat{V}_{\rm b} + \hat{V}_{\rm bf}, \tag{1}$$

$$\hat{H}_0 = \int d^3r \hat{\psi}^\dagger(\boldsymbol{r}) \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \frac{1}{2}m\omega_0^2 r^2 \right] \hat{\psi}(\boldsymbol{r}) + \int d^3r \hat{\phi}^\dagger(\boldsymbol{r}) \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + \frac{1}{2}m\omega_0^2 r^2 \right] \hat{\phi}(\boldsymbol{r}),$$

$$\hat{V}_{\rm b} = \frac{1}{2}g \iint d^3r d^3r' \hat{\phi}^\dagger(\boldsymbol{r}) \hat{\phi}^\dagger(\boldsymbol{r}') \delta^{(3)}(\boldsymbol{r}-\boldsymbol{r}') \hat{\phi}(\boldsymbol{r}') \hat{\phi}(\boldsymbol{r}),$$

$$\hat{V}_{\rm bf} = h \iint d^3r d^3r' \hat{\phi}^\dagger(\boldsymbol{r}) \hat{\psi}^\dagger(\boldsymbol{r}') \delta^{(3)}(\boldsymbol{r}-\boldsymbol{r}') \hat{\psi}(\boldsymbol{r}') \hat{\phi}(\boldsymbol{r}),$$

where $\hat{\phi}$ and $\hat{\psi}$ are the boson and the fermion field operators. The mass m and the trapping frequency ω_0 are taken to be the same for both kind of particles. The boson-boson and the boson-fermion interaction strengths for the pseudopotentials, g and h, are related to the s-wave scattering lengths $a_{\rm bb}$ and $a_{\rm bf}$ through

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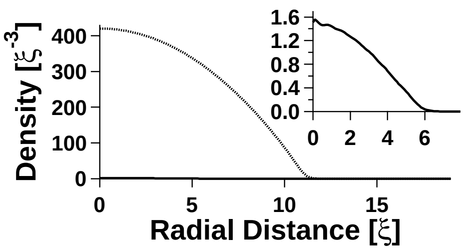

Fig. 1. Density distribution of bosons (dotted line) and fermions (insertion) for $h/g = 0.5$.

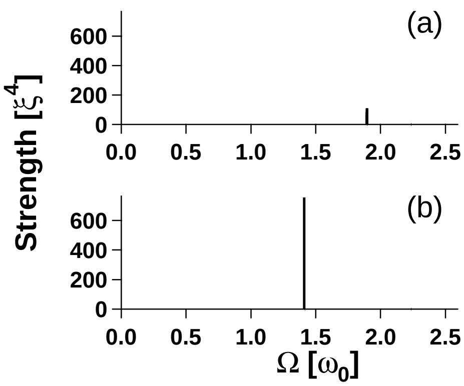

Fig. 2. Strength distribution for the fermionic external field of the monopole type F_0 is shown against excitation frequency Ω in unit of ω_0 for two values of the boson-fermion interaction a) $h/g = 0.1$ and b) 0.5.

$g = 4\pi\hbar^2 a_{\rm bb}/m$, $h = 4\pi\hbar^2 a_{\rm bf}/m$, while the fermion-fermion interaction is omitted because of the Pauli effect.

We first determine the mean field of bosons and fermions by solving a coupled system of Gross-Pitaevskii and Hartree-Fock equation. Based on the single-particle wave functions thus determined, an RPA equation is obtained for the system. The concrete form of the equation is similar to that in [2], where the RPA equation is derived for the bose-fermi mixed system based on the Thomas-Fermi type distribution.

We consider the mixed system of ^{41}K (boson) - ^{40}K (fermion) atoms with the mass $m = 0.649 \times 10^{-25}$kg. The scattering length for ^{41}K-^{41}K is fixed at $a_{\rm bb} = 15.13$nm, while the boson-fermion interaction strength h is varied. We take the trap frequency $\omega_0 = 1000$Hz which gives the oscillator length $\xi = \sqrt{\hbar/m\omega_0} = 1.27\mu$m for the parameters given above. Particle numbers of bosons and fermions are respectively taken as $N_{\rm b} = 10^6$ and $N_{\rm f} = 455$. For these particle numbers, the fermions are confined deep inside of bosons as shown in Fig. 1 for $h/g = 0.5$.

In Fig. 2(a) and (b) we show the strength distribution for the monopole external field $F_0 = r^2/\sqrt{4\pi}$ for fermions at $h/g = 0.1$ and 0.5 against excitation frequency Ω. In both cases the strength appears to be

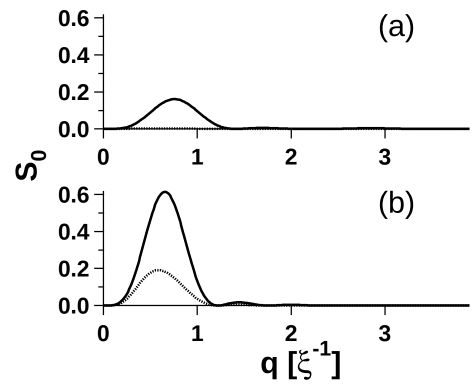

Fig. 3. Dynamical structure factors S_L are shown for the most collective monopole state for a) $h/g = 0.1$ and b)0.5. against momentum transfer q (unit of ξ^{-1}). Solid lines show the fermionic part and dotted lines the bosonic one.

concentrated in a narrow region. For $h/g = 0.1$, however, the peak is due to an incoherent sum of many states which is concentrated around $\Omega/\omega_0 \sim 1.9$ and carries only a fraction of the energy weighted sum $m_1 \simeq 801\hbar\omega_0\xi^4$. This is in contrast to the case of $h/g = 0.5$ where a single collective state exhausts almost 99% of the sum rule value $m_1 \simeq 1072\hbar\omega_0\xi^4$. We note also that the excitation frequency of the state is fairly low suggesting the softening of the monopole oscillation.

As there is no direct interaction between fermions, the coherent character of the fermionic collective state should be originate from the boson-fermion interaction. This can be checked by studying the dynamical structure factor $S_L(q, \Omega) = \sum_\nu |f_{\nu L}(q)|^2 \delta(\Omega - \Omega_\nu)$ with $f_{\nu L}(q) = \int {\rm d}r r^2 j_L(qr)\delta\rho_\nu(r)$, where $\delta\rho_\nu$ denotes the transition density for the excited state ν. This is shown in Fig. 3 for the most collective monopole states for $h/g = 0.1$ and 0.5 referred above. Evidently the collective state for $h/g = 0.5$ involves a fairly large contribution of bosons which shows a peak at the same value of q as that of fermions.

We conclude that a boson-fermion mixed system of atomic gases with a large excess of bosons may exhibit a fermionic collective monopole oscillation as a result of a fermion-fermion effective interaction mediated by bosons through a repulsive boson-fermion interaction. This kind of interaction is interesting also from the viewpoint of the fermionic superfluidity as discussed for a uniform bose-fermi system [3].

References

[1] A. G. Truscott *et. al.*, Science **291** (2001) 2570; F. Schreck *et. al.*, Phys. Rev. Lett. **87** (2001) 080403.

[2] T. Sogo *et. al.*, Phys. Rev. A (to be published); cond-mat/0202470.

[3] See, e.g., M. J. Bijlsma, B. A. Heringa, H. T. C. Stoof, Phys. Rev. A **61** (2000) 053601.