Diffusion of an Inhomogeneous Vortex Tangle

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Abstract

The spatial diffusion of an inhomogeneous vortex tangle is studied numerically with the vortex filament model. A localized initial tangle is prepared by applying a counterflow, and the tangle is allowed to diffuse freely after the counterflow is turned off. Comparison with the solution of a generalization of the Vinen equation that takes diffusion into account leads to a very small diffusion constant, as expected from simple theoretical considerations. The relevance of this result to recent experiments on the generation and decay of superfluid turbulence at very low temperatures is discussed.

Key words: superfluid turbulence; vortex tangle; helium4

1. Introduction

Recently there has been a growing interest in a comparison between classical turbulence and quantum turbulence in superfluid ^{4}He [1]. The ideal comparison requires experiments on homogeneous quantum turbulence produced by a grid towed at a steady velocity through the helium [2], and there is particular interest in the case of very low temperatures, when there is practically no normal fluid. Steady towing of a grid at these low temperatures leads to severe experimental problems, so the only existing relevant experiment used an oscillating grid instead of a towed grid [3]. Any interpretation of this experiment requires an understanding of the behaviour of inhomogeneous quantum turbulence, and this paper aims to provide relevant evidence, based on a computer simulation of the diffusion of a localized random vortex tangle. The results show that diffusion in this case is very slow, as is to be expected from simple dimensional considerations. Low-temperature grid turbulence may involve motion on scales much larger than that associated with a random tangle, which is of order the vortex-line spacing, in which case diffusion would be faster.

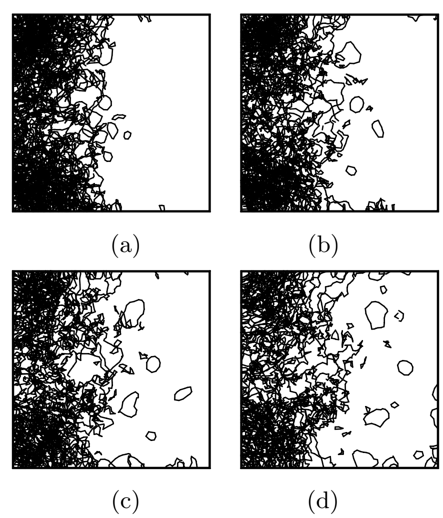

Fig. 1. Diffusion of a vortex tangle at t=0 sec(a), t=10.0 sec(b), t=20.0 sec(c) and t=30.0 sec(d).

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2. Numerical calculation

The required equation of motion of the vortices and the numerical procedures are described in detail in a previous paper [4]. The tangle is confined to a 1cm cube. An initial configuration of six vortex rings is allowed to evolve under a counterflow at 1.6K, using the local induction approximation. Periodic boundary conditions are applied at the faces normal to the flow; the other faces are taken as solid. When a random tangle has developed, the counterflow is turned off and the temperature reduced to zero, vortices with parts in the right-hand half of the cube are removed, and the evolution of the remaining vortices is followed, now with a fully non-local Biot-Savart dynamics. The evolution involves both decay and diffusion. The results are shown in Figs. 1 and 2.

3. A model equation to describe decay and diffusion of a vortex tangle

The decay of a homogeneous vortex tangle is described by the Vinen's equation

$$dL/dt = -\chi_2 \frac{\kappa}{2\pi} L^2, \tag{1}$$

where L is the vortex line density and κ is the quantum of circulation (h/m_4) [5]. The parameter χ_2 depends on temperature, being about 0.3 at zeto temperature [4]. A simple generalization of this equation for an inhomogeneous system can be obtained by adding a diffusion term. The resulting equation is unlikely to be rigorously correct, but it will serve for a preliminary analysis of the experiments results. That is, we assume that

$$\frac{dL(\boldsymbol{x},t)}{dt} = -\chi_2 \frac{\kappa}{2\pi} L(\boldsymbol{x},t)^2 + D\nabla^2 L(\boldsymbol{x},t). \tag{2}$$

where D is a diffusion coefficient, assumed constant for a given temperature. We see from the results in figure 2 that as time proceeds the line density falls near the wall at $x = 0$. This feature is not described by Eq. (2), and it is due presumably to an enhanced vortex decay rate in the neighbourhood of the wall. We take it into account by allowing χ_2 to increase close to the wall. As explained in Ref. [4], the simulations are based on a finite spatial resolution, Δx. The parameter χ_2 is therefore increased over the range from $x = 0$ to $x = \Delta x$ from the value χ_2 to a value $\chi_{2,B}$.

We have solved Eq. (2) numerically for the situation obtaining in the experiments. Taking $\chi_2 = 0.3$, and regarding D and $\chi_{2,B}$ as adjustable parameters, we have carried out least square fits to the experimental data. We find that at all times $D/\kappa = 0.1 \pm 0.05$ and $\chi_{2,D} = 0.7 \pm 0.05$.

An theoretical estimate of the likely value of D can be obtained as follows. We assume that the tangle is random, so that there is no significant superfluid motion on a scale larger than the line spacing $\ell = L^{-1/2}$. Then the only length scale relevant to the vortex motion is ℓ and the only velocity scale is κ/ℓ (we ignore logarithmic corrections arising from line curvature). It follows that D must be of order κ, as is observed.

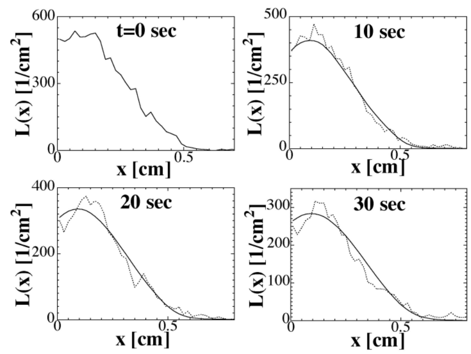

Fig. 2. Development of the vortex line density distribution. The dotted line shows the results of the numerical simulation of Fig. 1, while the solid lines are discussed in section 3.

4. Conclusion

The diffusion of an inhomogeneous random vortex tangle has been studied numerically with the vortex filament model. The diffusion can be described by a generalization of the Vinen equation, with a diffusion constant close to 0.1κ.

This result may be relevant to the generation of superfluid turbulence with an oscillating grid [3]. If the tangle that is produced has associated with it no motion on a length scale larger than the line spacing, then, as we see, the diffusion is very small, and the tangle can be expected to remain localized in the neighbourhood of the grid. If motion on a larger scale is generated, then the diffusion will be enhanced.

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