Superfluidity of Disordered Bose Systems: Numerical Analysis of the Gross-Pitaevskii Equation with Random Potential

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Abstract

We study the two-dimensional superfluidity of disordered Bose systems by analyzing the Gross-Pitaevskii equation with random potential. First, we obtain the ground state and calculate its superfluid density by the linear response theory. The superfluid density shows their remarkable dependence on the potential amplitude, the healing length and the density. Secondly, we apply the velocity field to the ground state to observe the breaking of superfluidity due to the excitation of vortex pairs above a critical velocity.

Key words: superfluidity; disorder; Gross-Pitaevskii equation; helium4

1. Introduction

Bose system in random environment is a significant problem for understanding not only the effect of disorder on its long-range order but also the long-range order itself, for example, the relation between Bose-Einstein condensation (BEC) and superfluidity. The experimental study of the problem has been made in liquid ^{4}He in porous Vycor glass[1], being proposed recently in alkali atomic BEC in a random optical trap[2]. There are a few theoretical studies like, for example, the Bose-Hubbard model[3]. This model describes the behavior of the amplitude of the macroscopic wave function of BEC in random environment, showing many interesting phenomena about dirty Bose system.

In this work, we study a two-dimensional disordered Bose system by using the Gross-Pitaevskii equation[4,5] with random potential. This model enables us to understand the effect of the random potential on both the amplitude and the phase of the macroscopic wave function, which lacks in the study with the Bose-Hubbard model. We can also investigate the dynamics of the system such as nucleation of vortex pairs. Our calculation leads to the new development of dirty Bose system.

2. The Gross-Pitaevskii model with random potential

The macroscopic wave function for interacting Bose condensation in the random external potential at the zero temperature satisfies the Gross-Pitavskii equation

$$(\mathrm{i}-\gamma)\frac{\partial\Phi(\boldsymbol{x})}{\partial t} = \Big[-\frac{\hbar^2\nabla^2}{2m} - \mu + \boldsymbol{v}\cdot\boldsymbol{\nabla} + V(\boldsymbol{x}) + g|\Phi(\boldsymbol{x})|^2\Big]\Phi(\boldsymbol{x}), \quad (1)$$

where $\Phi(\boldsymbol{x})$ is the macroscopic wave function, m is the mass of a boson, μ is the chemical potential, g is the coupling constant of the interaction between bosons, $\boldsymbol{v}$ is the external velocity field and γ is a dissipation coefficient. We solve numerically this equation for two-dimensional systems. $V(\boldsymbol{x})$ is the external random potential. We take the ensemble average with respect to the random potentials whose peaks have the same characteristic strength and width. The typical random potential is shown in Fig. 1(a). For one random poten-

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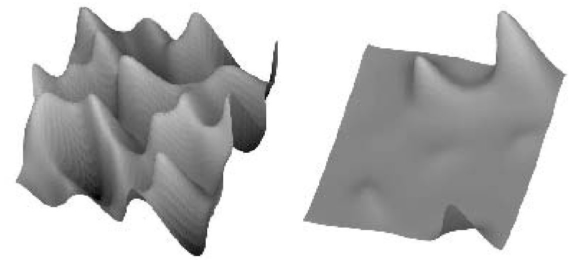
tial, we can obtain the corresponding ground state by finding the steady state of the time dependent Eq. (1) with $\gamma \neq 0$. Figure 1(b) is the amplitude of the ground state for the potential of Fig. 1(a). The condensate is trapped in the valleys whose widths are larger than the healing length $\xi = \sqrt{\hbar^2/2m\mu}$ among many valleys of the random potential.

We can calculate the superfluid density of the ground state by the linear response theory[6]. The superfluid density is sensitive to the amplitude of the potential, the healing length ξ and the particle density. Figure 2 shows a dependence of the superfluid density on the healing length. As ξ is reduced below a critical value, the superfluid density becomes rapidly small. The short healing length makes the condensate localize in the valleys and prevents its extension over the system, thus depressing the superfluidity

When a velocity field $\boldsymbol{v}$ is applied, the ground state makes characteristic nonlinear response, such as nucleating vortex pairs above an critical velocity field. Figure 3(a) and (b) shows the amplitude and phase, respectively, when a vortex pair is nucleated. In Fig. 3(a), the amplitude becomes very small in the black region, and in Fig. 3(b), the value of the phase varies continuously from $-\pi$(black) to π(white). The phase changes by 2π around the points shown by arrows; these points are vortices, where the density is very small in Fig. 3(a) The vortices move along the region with the small amplitude. The appearance of vortices complicates the space structure of the phase, thus suppressing the superfluidity considerably.

The detailed studies of this system are now in progress.



(a) (b)

Fig. 1. One sample of random potential (a) and the ground state (b) for the potential. Characteristic strength of the random potential is $\bar{V} = 25\mu$, where $\bar{V}$ is the spatial averaged value of $V(\boldsymbol{x})$. The healing length is $\xi = \lambda$, where λ is the characteristic width of the random potential.

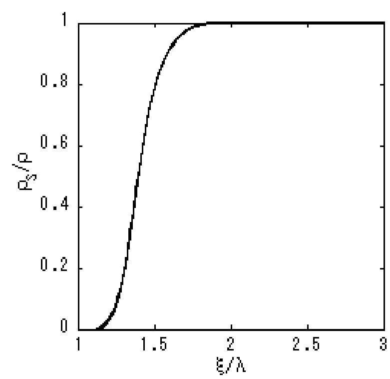
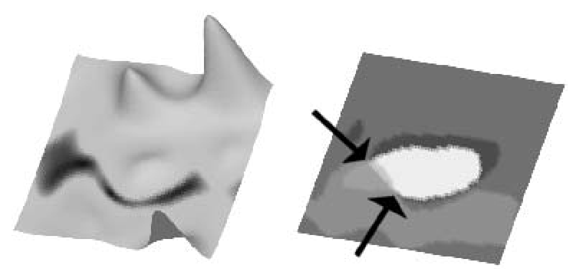


Fig. 2. The healing length dependence of the superfluid density. ρ is the density and ρ_s is the superfluid density. We take one hundred ensemble average for the same strength $\bar{V}$ of the random potential.



(a) (b)

Fig. 3. The amplitude(a) and phase(b) of the wave function at $t = 2.75\hbar/\mu$ and $\boldsymbol{v} = 0.8\sqrt{\mu/2m}$, where initial $t = 0$ condition is Fig. 1(b). In (a), the amplitude becomes very small in the black region. In (b), the value of the phase varies continuously from $-\pi$(black) to π(white).

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