Interlayer Josephson coupling in layered superconductors

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Abstract

The dependence of the Josephson interlayer coupling in layered superconductors on the magnetic field H is studied numerically in the limit of complete disorder of the positions of pancake vortices (pancake gas). We find that the spatial average $\langle\cos\varphi(\mathbf{r})\rangle$ is proportional to $1/H^{1/2}$, where φ is the gauge-invariant phase difference between two layers. This result is compared with the magnetoabsorption resonances observed in layered superconductors.

Key words: layered superconductors; pancakes; magnetoabsorption resonance; Josephson plasma resonance

It is widely believed that magnetoabsorption microwave resonances, which have been observed in Bi compounds and some other layered superconductors, are related to the Josephson Plasma Resonance (JPR) with the frequency (see e.g. [1] and references therein)

$$\omega_0^2 = \omega_p^2\langle\cos\varphi(\mathbf{r})\rangle\ , \tag{1}$$

where ω_p is the Josephson-plasma frequency at zero magnetic field, $\mathbf{r} = (x, y)$ is the inplane coordinate, $\varphi(\mathbf{r})$ is the stationary gauge-invariant phase difference between neighboring layers, and $\langle\ldots\rangle$ denotes spatial averaging. If the magnetic field H normal to the layers generates ideally straight vortices one has $\varphi(\mathbf{r}) = 0$, and the resonance frequency does not depend on the magnetic field. But one has $\varphi(\mathbf{r}) \neq 0$ if there is a misalignment of the pancake vortices in neighboring layers due to thermal fluctuations and disorder.

A characteristic feature of the observed magnetoabsorption resonances at high magnetic fields is the anticyclotronic behavior: $\omega_0^2 \propto 1/H^\alpha$, where α was reported to be between 0.7 and 1 (see [1] and references therein). In order to check whether this behavior follows from Eq. (1) we have calculated numerically $\langle\cos\varphi(\mathbf{r})\rangle$ for a gas of pancake vortices, in which disorder destroys any correlation of pancake positions both between and inside the layers.

In principle, we should solve the system of stationary sine-Gordon equations for the phases across all interlayer spacings. But one may expect to receive a correct physical picture by solving the simpler problem of a double layer with only one fluctuating phase $\varphi(x, y)$, which is described by the sine-Gordon equation:

$$\frac{1}{\lambda_J^2}\sin\varphi - \nabla^2\varphi = 0\,. \tag{2}$$

The pancakes in the upper and lower layer may be considered as vortices and antivortices, projected unto the x, y plane, with $\varphi(x, y)$ now being the phase of this 2D arrangement of vortex–antivortex pairs.

At high fields the intervortex distance $a = \sqrt{\Phi_0/B}$ is much less than the Josephson length λ_J. Neglecting the Josephson coupling $\propto 1/\lambda_J^2$ in Eq. (2), two pancakes in the neighboring layers at the points (0,0) and $(0,r_w)$ generate the phase φ (short Josephson string):

$$\varphi_0(\mathbf{r}) = \arctan\frac{y}{x} - \arctan\frac{y}{x - r_w}\,. \tag{3}$$

At large distances $r = \sqrt{x^2 + y^2} \gg r_w$ the phase is $\varphi_0(\mathbf{r}) = -r_w y/r^2$. This means that the contribution of one Josephson string to $\langle\varphi_0(r)^2\rangle$ is logarithmically divergent with the system size, and we cannot neglect the Josephson coupling even in the limit $\lambda_J \gg a$. But one may hope that the effect of finite λ_J can be simu-

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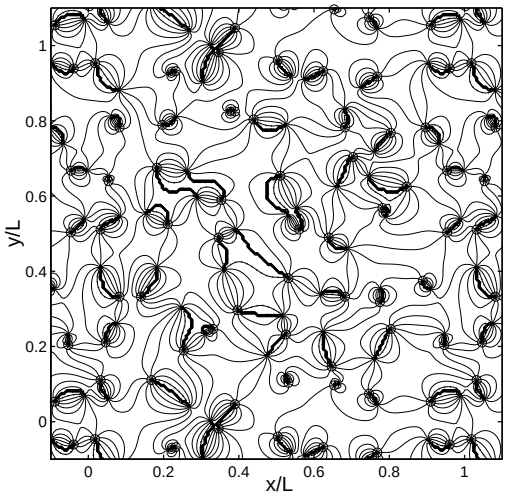

Fig. 1. Lines of equal phase φ for 49 vortex–antivortex pairs shifted from ideal square lattice positions with spacing a by random displacements with variance $s/a = 0.25$. The variance of the pair spacing is $\sqrt{2}s$. The contour spacing is $\pi/6$. The jump of the phase from $+\pi$ to $-\pi$ may be chosen at the bold lines; the lines with dots then shown $\varphi = 5\pi/6$.

lated by choosing a finite size $L \approx \lambda_J$ of the considered area with a large number of vortex-antivortex pairs.

In our numerical calculation we choose more or less random positions of N vortices and N antivortices in a quadratic cell of the size $L \times L$, so the average distance of each type is $a = L/\sqrt{N}$. The phase $\varphi(x, y)$ is the solution of Eq. (2) at $\lambda_J \to \infty$ (the Laplace equation):

$$\varphi(x,y) + 2\pi n = \arg \prod_1^N \frac{z - z_i^v}{z - z_i^a}$$

$$= \sum_1^N \left[\arctan \frac{y - y_i^v}{x - x_i^v} - \arctan \frac{y - y_i^a}{x - x_i^a} \right], \qquad (4)$$

where n is an integer, (x_i^v, y_i^v) and (x_i^a, y_i^a) are the positions of the vortices and antivortices, respectively, and $z = x + iy$ etc. are complex numbers.

To simulate disorder, we shift each vortex and each antivortex by a Gaussian distributed random vector of mean square length s away from its position in a perfect square lattice ($\varphi = 0$ at $s = 0$). Figure 1 shows the equal-phase lines for an arrangement of 49 vortex-antivortex pairs with relative displacement amplitude $s/a = 0.25$.

In our simulations we calculate

$$f = \langle \cos[\varphi(x,y) - \varphi_0] \rangle_{\max} = |\langle \exp[i\varphi(x,y)] \rangle| \,, \qquad (5)$$

which determines the frequency: $\omega_0^2 = \omega_p^2 f$. The factor f depends on the degree of randomness, thus $f = f(s)$. With increasing displacement amplitude s the phase fluctuations increase, and when s/a exceeds the value ≈ 0.5, we find saturation to the limit of completely

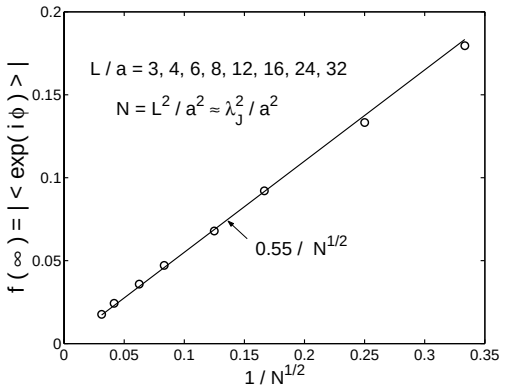

Fig. 2. The spatial average $f(\infty) = | < \exp(i\varphi) > |$ for a gas of randomly positioned vortices and antivortices as a function of the number of pairs in the periodicity cell, $N = L^2/a^2 \approx \lambda_J^2/a^2$. The linear fit shows $f(\infty) = 0.55/N^{1/2}$.

uncorrelated randomly positioned N vortices and N antivortices. Choosing various boundary conditions at the border of the square $L \times L$, we find that the treatment of the vortices near the boundaries is not crucial.

The pancake gas corresponds to the limit of completely random positions, $s \approx r_w \gg a$, when anticyclotronic behavior is expected. We find that $\langle \cos \varphi(\mathbf{r}) \rangle_{\max} = f(\infty) = c \cdot a/L$ with the constant $c = 0.55$. (Fig. 2). Since L has the meaning of the Josephson length λ_J, different N correspond to different ratios $a/\lambda_J \approx a/L = N^{-1/2}$. On the other hand the number of pancakes $N = L^2/a^2$ in our cell is proportional to the magnetic field $H = \Phi_0/a^2$, where a is the intervortex distance.

Thus our numerical calculation yields anticyclotronic power-law behavior $\omega_0^2 \propto 1/H^\alpha$ with an exponent $\alpha = 1/2$. This clearly differs from the exponent $\alpha = 0.7 \div 1$ deduced from the experiment. A possible way to resolve this disagreement is to consider other interpretations of the magnetoabsorption resonances at high normal magnetic fields, which are not based on JPR and Eq. (1) (see discussion in [2,3]).

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