Non-universal critical Casimir force in confined ^{4}He near the superfluid transition

X.S. Chen [a,b], V. Dohm [b]

[a] *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China*
[b] *Institut für Theoretische Physik, Technische Hochschule Aachen, D-52056 Aachen, Germany*

Abstract

We present the results of a one-loop calculation of the effect of a van der Waals type interaction potential $\sim |\mathbf{x}|^{-d-\sigma}$ on the critical Casimir force and specific heat of confined ^{4}He near the superfluid transition. We consider a ^{4}He film of thickness L. In the region $L \gtrsim \xi$ (correlation length) we find that the van der Waals interaction causes a leading non-universal non-scaling contribution of $O(\xi^2 L^{-d-\sigma})$ to the critical temperature dependence of the Casimir force above T_λ that dominates the universal scaling contribution $\sim e^{-L/\xi}$ predicted by earlier theories. For the specific heat we find subleading non-scaling contributions of $O(L^{-1})$ and $O(L^{-d-\sigma})$.

Key words: critical phenomena, finite-size effects, helium4, Casimir force

In the last decade the critical Casimir effect in ^{4}He films has been an interesting topic of theoretical [1,2,3] and experimental [4] research. On the basis of field-theoretic calculations for purely short-range interaction it was predicted that the critical temperature dependence of the Casimir force is governed by a universal scaling function $X(L/\xi)$ where L is the film thickness and $\xi = \xi_0 t^{-\nu}$, $t = (T - T_\lambda)/T_\lambda$, is the bulk correlation length. The universal behavior for large L/ξ was predicted to be $\sim e^{-L/\xi}$. Here we investigate the question which finite-size effect arises from van der Waals type interactions. They do not change the leading universal *bulk* critical behavior of the free energy and specific heat. The finite-size effects of these interactions were discussed in Refs. [1,2,3] where it was assumed that they yield only *non-singular* (temperature-independent) contributions. Here we shall show that these interactions yield a leading *singular* non-scaling temperature dependence of the Casimir force that is non-universal and dominates the exponential scaling contribution above T_λ for large L/ξ. We also present predictions for subleading non-scaling contributions to the critical specific heat of ^{4}He films above T_λ that may be non-negligible in the analysis of experimental data.

The present paper is a continuation of recent work where leading non-scaling cutoff and lattice effects [5,6,7,8] and non-scaling effects of van der Waals type interactions [8,9,10] have been found both for the finite-size susceptibility and for the bulk order-parameter correlation function. These effects were not taken into account in the previous theory [1,2,3] that was based on the usual φ^4 Hamiltonian in d dimensions

$$H\{\varphi\} = \int d^d x \left[\frac{1}{2} r_0 \varphi^2 + \frac{1}{2}(\nabla\varphi)^2 + u_0 (\varphi^2)^2\right] \qquad (1)$$

for the n component field $\varphi(\mathbf{x})$. Here the short-range interaction term $(\nabla\varphi)^2$ has the Fourier representation $\mathbf{k}^2 \varphi_\mathbf{k} \varphi_{-\mathbf{k}}$. In the following we assume the existence of an additional subleading long-range interaction term $b|\mathbf{k}|^\sigma \varphi_\mathbf{k} \varphi_{-\mathbf{k}}$ with $2 < \sigma < 4$ which corresponds to a van der Waals type spatial interaction potential $U(\mathbf{x}) \sim |\mathbf{x}|^{-d-\sigma}$. For the case of ^{4}He ($n = 2$) we shall consider film geometry with Dirichlet boundary conditions.

We have performed a one-loop calculation of the critical Casimir force $F = F_s + F_{ns}$. For the singular part F_s we have found

$$F_s(\xi, L, b) = -bL^{-d+2-\sigma} B(L/\xi) + L^{-d} X(L/\xi) \qquad (2)$$

where the last term $\sim L^{-d}$ is the universal scaling part due to the short-range interaction. For $\xi \gg L$ this scaling part is dominant. For large L/ξ, however, it becomes exponentially small, $X(L/\xi) \sim e^{-L/\xi}$ [1].

The non-universal non-scaling term $\sim b$ due to the van der Waals type interaction reads for $L/\xi \gtrsim 1$

$$B(L/\xi) = (d - 3 + \sigma)\Psi(L/\xi) - (L/\xi)\Psi'(L/\xi) \ , \quad (3)$$

$$\Psi(L/\xi) = \frac{1}{2}(2\pi)^{\sigma-4} \int\limits_{(L/\xi)^2}^{\infty} dx \left(1 + x\frac{\partial}{\partial x}\right) \widetilde{\Psi}(x) \ , \quad (4)$$

$$\widetilde{\Psi}(x) = \int\limits_0^{\infty} dy \ \ y^{(2-\sigma)/2} e^{-xy/4\pi^2} \left(\sqrt{\frac{\pi}{y}}\right)^{d-1} \widetilde{W}_1(y) \times \\ \times \gamma^* \left(\frac{2-\sigma}{2} \ , -\frac{xy}{4\pi^2}\right) \quad (5)$$

where $\gamma^*(z,x) = x^{-z} \ \int_0^x \ dte^{-t} \ t^{z-1} / \int_0^\infty \ dte^{-t} \ t^{z-1}$ is the incomplete Gamma function and

$$\widetilde{W}_1(y) = \sqrt{\frac{\pi}{y}} - \frac{1}{2}\sum_{n=-\infty}^{\infty} \exp\left(-\frac{y}{4}n^2\right) . \quad (6)$$

The large L/ξ behavior is $\Psi(L/\xi) \sim \xi^2 L^{-2}$. Thus the leading critical temperature dependence $F_s(\xi, L, b) \sim b\xi^2 L^{-d-\sigma}$ is nonuniversal for $L \gtrsim \xi$ and dominates the exponentially small scaling part.

Corresponding non-universal non-scaling effects exist for the free energy [8] which implies a significant restriction for the range of validity of the universal Privman-Fisher scaling form [11,12].

We have also calculated the finite-size effect of the van der Waals interaction on the specific heat in one-loop order for $L \gtrsim \xi$ above T_λ. The result reads

$$C(\xi, L, b) = C_b + 2C_{surface}\left[1 + b\widetilde{A}(\sigma,d)\xi^{2-\sigma}\right] L^{-1} \\ + \ \Delta C + 2ba_0^2 \widetilde{B}(\sigma,d)\xi^6 L^{-\sigma-d} \quad (7)$$

with $a_0 = (r_0 - r_{0c})/t$ where C_b, $C_{surface}$ and $\Delta C = L^{\alpha/\nu} g(L/\xi)$ are the known bulk, surface and finite-size parts of the specific heat due to the short range interaction [13]. For the relation between a_0 and ξ_0 see [14]. The amplitudes of the non-universal non-scaling terms $\sim b$ due to the van der Waals type interaction are

$$\widetilde{A}(\sigma,d) = -\Gamma\left(\frac{5-d}{2}\right)^{-1} \int\limits_0^{\infty} ds \ s^{(3-d-\sigma)/2} e^{-s} \tilde{f}_\sigma(s) \quad (8)$$

with

$$\tilde{f}_\sigma(s) = \frac{1}{4}\sigma(\sigma-2)\gamma^*\left(\frac{2-\sigma}{2}, -s\right) + \sigma\gamma^*\left(-\frac{\sigma}{2}, -s\right) \\ + \gamma^*\left(-1 - \frac{\sigma}{2}, -s\right) \quad (9)$$

and

$$\widetilde{B}(\sigma,d) = -\frac{\sigma(\sigma-2)}{2^{d+2}\pi^{d/2}} \frac{\Gamma\left(\frac{d+\sigma}{2}\right)}{\Gamma\left(\frac{4-\sigma}{2}\right)} \zeta(d+\sigma) \quad (10)$$

where ζ is Riemann's zeta function.

The next step of our theory will be to specify the nonuniversal strength b and the exponent σ of the van der Waals type interaction in ^{4}He. Since these parameters are as yet unknown the range of applicability of the earlier predictions for the universal finite-size scaling functions of the Casimir force and of the specific heat above T_λ is not yet established.

Finally we note that our one-loop results $\sim b$ were obtained by a calculation at the Gaussian level (with $u_0 = 0$, $r_{0c} = 0$ and at infinite cutoff) where part of the non-Gaussian (higher-loop) effects were taken into account by the replacement $r_0 \to \xi^{-2}$. It remains to be seen to what extent a full renormalization-group treatment with $u_0 > 0$ at the two-loop level confirms this procedure.In this sense our predictions of the temperature dependence of the terms $\sim b$ in Eqs. (2) and (7) should be considered only as preliminary. A full renormalization-group treatment will be given elsewhere.

Acknowledgements

We acknowledge support by DLR and by NASA under grant numbers 50WM9911 and 1226553.

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