Effect of orbital ferromagnetism on textures in a slab of ^{3}He-A

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Abstract

The effect of orbital ferromagnetism on the Fréedericksz transition of a uniform **l**-texture of ^{3}He-A in a slab geometry has been calculated. The transition magnetic field is altered in opposite directions for **l** parallel and antiparallel to the field by an amount proportional to the ferromagnetic moment. At high fields, for **l** initially antiparallel to the field we find another threshold above which the uniform **l**-texture is restored with **d** perpendicular to **l**.

Key words: superfluid; helium-3; texture; ferromagnetism

1. Introduction

The A phase of superfluid ^{3}He is predicted to have an orbital magnetic moment of the order of 10^{-11} μ_B per atom antiparallel to the orbital anisotropy vector **l** [1], making ^{3}He-A a ferromagnetic liquid. There has been one reported observation of the orbital ferromagnetism [2] but the cell geometry resulted in non-uniform textures making any quantitative estimate of the effect difficult. A slab geometry is ideally suited for detecting the influence of orbital ferromagnetism on the texture since uniform **l**-textures can be obtained and manipulated [3]. There have been several previous studies of ^{3}He-A in a slab (see [4] for a review).

We consider a sample of ^{3}He-A contained between two infinite parallel plates at $z = \pm D/2$ and contained in the xy plane with an applied field **H** in the z direction. The texture is characterized by two angles: $\theta(z)$ is the angle between **l** and $\hat{\mathbf{z}}$ and $\phi(z)$ is the angle between **d** and $\hat{\mathbf{z}}$. The free-energy density is

$$F = \frac{1}{2}(K_S \sin^2\theta + K_B \cos^2\theta)\left(\frac{d\theta}{dz}\right)^2 + \frac{1}{2}(\rho_{sp\perp}\sin^2\theta + \rho_{sp\parallel}\cos^2\theta)\left(\frac{d\phi}{dz}\right)^2 - \frac{1}{2}\lambda_D\cos^2(\theta-\phi) + \frac{1}{2}\Delta\chi H^2\cos^2\phi - M_l\, H\cos\theta \qquad (1)$$

where $\mathbf{M_l}$ is the orbital magnetization and the other coefficients follow the notation of [4]. The free energy has been minimized by using the calculus of variations to find Euler equations for θ and ϕ. The texture is then found by solving these equations subject to the boundary conditions that $\theta = 0$ and $d\phi/dz = 0$ at $z = \pm D/2$. The solutions discussed in the following sections use the following dimensionless parameters: $D/L_D = 25$, $K_B/\rho_{sp\parallel} = 1.5$, $K_S/\rho_{sp\parallel} = 0.5$, $\rho_{sp\perp}/\rho_{sp\parallel} = 2$ and $M_l/(\Delta\chi H_D) = -10^{-4}$ where $L_D = (\rho_{sp\parallel}/\lambda_D)^{1/2}$ is the dipole-unlocking length ($\sim 10\,\mu$m). The applied magnetic field is given in units of $H_D = (\lambda_D/\Delta\chi)^{1/2}$, the dipole-unlocking field (~ 25 Gauss).

2. Low field behaviour

Previous studies have shown that for $H < H_F$ the stable solution is the uniform texture ($\theta = \phi = 0$). When $H \geq H_F$ the texture in the centre of the slab becomes distorted. This textural transition is known as a Fréedericksz transition and has been measured for a slab of ^{3}He-A [5]. The effect of orbital ferromagnetism is to cause a difference in the threshold field between the states with **l** initially parallel and antiparallel to

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H. In the dipole-locked limit ($\theta = \phi$ everywhere) the threshold field is modified,

$$H_F = H_{F0} \pm \frac{M_l}{2\Delta\chi} \tag{2}$$

where H_{F0} is the threshold in the absence of orbital ferromagnetism. The different signs in Eq. 2 correspond to the states where **l** is parallel (-) and antiparallel (+) to **H**. The difference is $\Delta H_F = M_l/\Delta\chi \sim 2\times10^{-3}$ Gauss, which should be independent of temperature to first order. We have calculated textures including dipole-unlocking effects numerically for $H > H_F$. Fig. 1 shows the angle between **l** and $\hat{\mathbf{z}}$ in the centre of the slab, $\theta(0)$, for different values of applied field near H_{F0}.

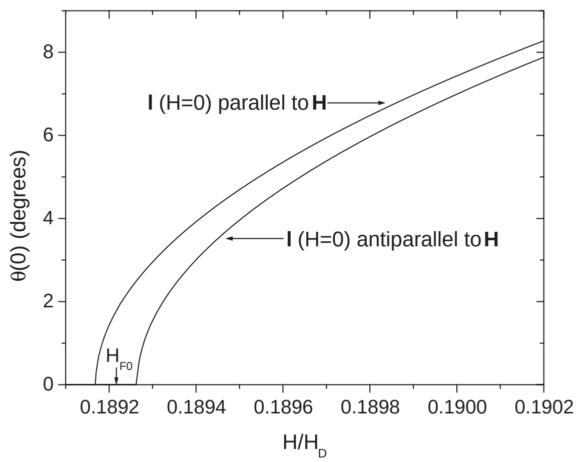

Fig. 1. Effect of orbital ferromagnetism on Fréedericksz transition field.

3. High field behaviour

In the absence of orbital ferromagnetism both $\theta(0)$ and $\phi(0)$ tend asymptotically towards $\pi/2$ as magnetic field is increased. The effect of orbital ferromagnetism is that as magnetic field increases **d** remains perpendicular to **H** but the amount of dipole-unlocking $(\theta-\phi)$ increases as **l** tries to align itself antiparallel to **H** (Fig. 2). In this case the **l**-texture in the centre of the slab is approximately

$$\theta(0) \simeq \frac{\pi}{2} \pm \frac{M_l\, H}{\lambda_D} \tag{3}$$

where the difference in sign indicates **l** initially antiparallel (-) and parallel (+) to **H**.

In the former case a uniform texture with $\theta(z) = 0$ and $\phi(z) = \pi/2$ occurs after the magnetic field reaches a threshold value

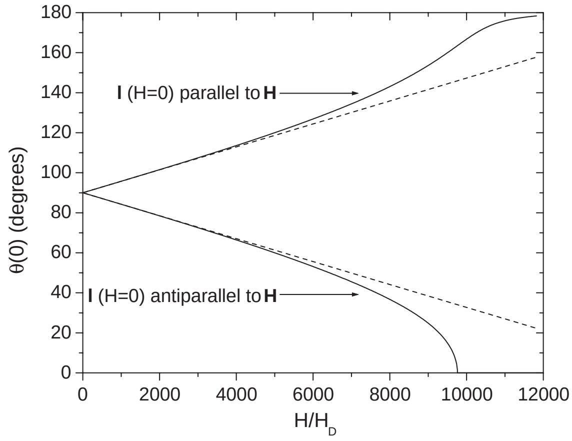

Fig. 2. Angle between **l** and $\hat{\mathbf{z}}$ at high fields. The dashed lines show the approximate solutions given by Eq. 3. The low field region (Fig. 1) cannot be resolved.

$$H_c = \frac{1}{M_l}\left(\lambda_D - K_B\frac{\pi^2}{D^2}\right) \tag{4}$$

which is 9763 H_D for the parameters given in section 1. In the latter case, the **l**-texture in the centre becomes aligned in the opposite direction to the texture at the slab boundary resulting in a higher free-energy compared to the previous case. At certain values of magnetic field this metastable state should change to the ground state due to either an extrinsic process (growth of a domain of the alternative state from a nucleated or pinned defect such as a vortex) or an intrinsic process (a global instability).

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