Full counting statistics of a superconducting beam splitter

Wolfgang Belzig [a,1], Johannes Börlin [a], Christoph Bruder [a]

[a] *Departement für Physik und Astronomie, Klingelbergstr. 82, 4056 Basel, Switzerland*

Abstract

The complete information about a charge transfer process is contained in the so-called *full counting statistics*. We study the statistics of charge transport in a mesoscopic three-terminal device with one superconducting terminal and two normal-metal terminals. We calculate the full distribution of transmitted charges into the two symmetrically biased normal terminals. On one hand, in limit of weak proximity we find positive crosscorrelations and show that this is due to splitting of uncorrelated Cooper pairs. On the other hand, correlated Cooper pairs lead to negative crosscorrelation, as usually expected for a fermionic system.

Key words: shot noise, proximity effect, crosscorrelation

1. Introduction

The number of charges transfered in a transport process fluctuates due to quantum-mechanical uncertainty and statistics. Therefore, the outcome of a current measurement accumulated over some time period t_0 is in general described by a probability $P(N)$, where N is the total number of charges transfered. $P(N)$ is called the *full counting statistics* (FCS) of the transport process [1]. The first two moments of the FCS are related to the average current and the current noise and are accessible to present experimental techniques. Higher-order correlations are likely to be measured in the future.

The current noise, i. e., the second moment of the FCS, is of particular interest. For superconductor(S)-normal metal(N) heterostructures, a doubling of the shot noise in comparison to the normal case was predicted and measured in diffusive heterostructures. So far, crosscorrelations, i. e., current correlations involving different terminals, were measured only in normal single-channel heterostructures. These have confirmed that current crosscorrelations in a fermionic system are always negative [2]. To our knowledge, there is no measurement of crosscorrelations in a system with superconducting contacts up to now.

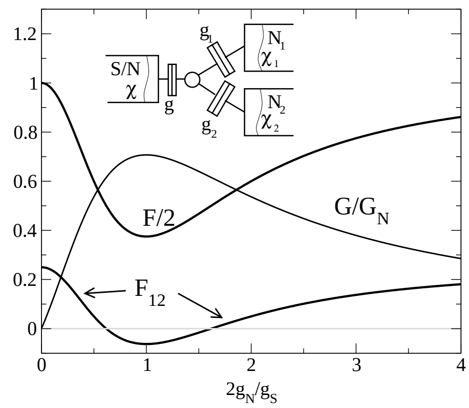

Fig. 1. Transport properties of the supercondcting beam splitter depicted in the inset.

The setup of our three-terminal device with one superconducting (S) and two normal-metal (N) terminals is shown in the inset of Fig. 1. All three terminals are connected by tunnel junctions to a small normal-metal island. Using an extended Keldysh Green's function approach[3,4], we found the FCS of the beam splitter as detailed in Ref. [5] for the case, in which no current flows between the normal

[1] Corresponding author. E-mail: Wolfgang.Belzig@unibas.ch

terminals. The probability $P(N_1, N_2)$, that $N_1(N_2)$ electrons are transported in terminal N1(N2), is represented by its cumulant generating function $S = \ln\left[\sum_{N_1,N_2} P(N_1,N_2)\exp(iN_1\chi_1 + iN_2\chi_2)\right]$. Introducing $p_i = 2gg_i/(g^2+(g_1+g_2)^2)$ we find

$$S = M\sqrt{1+\sqrt{1+(p_1e^{i\chi_1}+p_2e^{i\chi_2})^2-(p_1+p_2)^2}}\,,$$

where $M = Vt_0((g^2+(g_1+g_2)^2)/2)^{1/2}/e$. This result for the cumulant-generating function incorporates all statistical transport properties for our present setup. The inner argument contains counting factors for the different possible processes. A term $\exp(i(\chi_k+\chi_l)-1)$ corresponds to an event in which two charges leave the superconducting terminal and one charge is counted in terminal k and one charge in terminal l. The prefactors are related to the corresponding probabilities. The double square-root function shows that these different processes are non-separable.

We will now concentrate on auto- and crosscorrelations only. We obtain with $G = g_S^2 g_N^2/(g_S^2+g_N^2)^{3/2}$ and defining $F = S/2eI$, $F_{12} = S_{12}/2eI$ and $p = p_1 + p_2$

$$F = 2 - \frac{5}{4}p^2\,,\; F_{12} = \frac{g_1g_2}{(g_1+g_2)^2}\left(1-\frac{5}{4}p^2\right)$$

The conductance vanishes rapidly in the limit of weak proximity and shows a resonance in the regime of optimal proximity. This actually defines the condition for optimal proximity effect: the regime, in which the proximity effect has the largest impact on the conductance. Note, that without proximity effect the conductance would vanish always in the present geometry. Around the resonance condition $g_S \approx g_N$ the Fano factor drops below 2 and reaches a minimum of 3/4 for $g_S = g_N$. The most surprising observation is that we obtain positive crosscorrelations in the superconducting beam splitter. Interestingly the positive crosscorrelations are found in the limit of weak proximity. The FCS in this regime was obtained in [5]

$$P(N_1,N_2) = \frac{e^{-\frac{\bar N}{2}}\left(\frac{\bar N}{2}\right)^{\frac{N_1+N_2}{2}}}{\left(\frac{N_1+N_2}{2}\right)!}\begin{pmatrix} N_1+N_2 \\ N_1 \end{pmatrix} T_1^{N_1}T_2^{N_2}\,.$$

Here we have defined the average number of transfered electrons $\bar N = t_0 G^S V/e$ and the probabilities $T_{1(2)} = g_{1(2)}/(g_1+g_2)$ that one electron leaves the island into terminal 1(2). The possible events are pair tunneling into terminal N_1 or N_2 and simultanous tunneling into both terminals. These events occur with equal probabilities, however, in the limit of weak proximity these events are *uncorrelated*. The distribution is given by a Poisson distribution of tunneled pairs, multiplied with a *partitioning factor*, which corresponds to the number of ways how $N_1 + N_2$ identical electrons can be distributed among the terminals 1 and 2, with respective probabilities T_1 and T_2.

Following this argumentation to the regime of optimal proximity effect in fact shows, that it is much harder to understand the occurrence of *negative* correlations in this regime. We propose the following interpretation. The suppression of the Fano factor suggests that transfer of pairs is correlated. An extreme case would be a *quite* transfer. The corresponding statistics would be

$$P(N_1,N_2) = \delta_{M,N_1+N_2}\begin{pmatrix} N_1+N_2 \\ N_1 \end{pmatrix} T_1^{N_1}T_2^{N_2}\,.$$

Here M denotes the number of impinging Cooper pairs and we have neglected any back reflection. It is easy to show that this leads to $F_{12} = -1/2$, i.e. negative crosscorrelations.

The origin of the antibunching of the incoming beam can be understood as follows. As a consequence of the Pauli principle, that two fermions can not occupy the same state. Now, the same holds obviously for pairs of electrons. If a two particle state is occupied by two electrons, this state is blocked for other pairs of electrons. Cooper pairs are such objects, and can not occupy the same state twice. Just as in the case of Fermions, this leads to a strong suppression of the Fano factor and consequently to negative crosscorrelations.

In conclusion we have considered the full counting statistics of a superconducting beam splitter. We have shown that the sign of the cross correlations between the currents in the normal metal depends crucially on the statistics of the beam of Cooper pairs, which is split. Uncorrelated pair transfer results in positive crosscorrelations. Antibunching of the Cooper pairs occurs because they consist of two electrons and suppressed the noise of the beam of Cooper pairs. This finally results in negative crosscorrelations of the electrons in the two normal metals. Since these arguments do not depend on details of the setup, we believe an experimental observation of the positive cross correlations is feasible with standard technology.

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