Weak Coupling Approach to Chirality-driven Anomalous Hall Effect

Gen Tatara [a,1], Hikaru Kawamura [a]

[a] *Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan*

Abstract

Anomalous Hall effect arising from non-trivial spin configuration (chirality) is studied based on the *s*-*d* model in the weak coupling case. Chirality is shown to drive locally Hall current and leads to overall Hall effect if there is a finite uniform chirality. This contribution is independent of the conventional spin-orbit contribution and shows distinct low temperature behavior. Measurement of Hall coefficient would be useful in experimentally confirming the chirality ordering in various frustrated systems.

Key words: Hall effect; chirality; frustration; Berry phase

Recently it was found that some manganites and ferromagnetic pyrochlores exhibit abnormal Anomalous Hall effect[1,4]. The behavior was explained by Berry phase effect associated with non-trivial background spin configuration (chirality) driven thermally or by geometrical frustration[4,2,3]. These theories have exclusively dealt with a strong Hund-coupling limit, considering a half-metallic nature of the experimental systems. In this paper, we study anomalous Hall effect due to chirality based on the single-band *s*-*d* model in the weak coupling case. Using Kubo formula and taking account of the impurity scattering, we will demonstrate that the chirality drives local Hall current in the perturbative regime.

We consider electron whose Hamiltonian is given by

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{J}{N} \sum_{\mathbf{k}\mathbf{k}'} \mathbf{S}_{\mathbf{k}'-\mathbf{k}} (c^{\dagger}_{\mathbf{k}'} \sigma c_{\mathbf{k}}), \quad (1)$$

where $\sigma = \pm$ denotes electron spin. Electron energy is $\epsilon_{\mathbf{k}}$. The Zeeman splitting due to magnetization is neglected. Configuration of localized spin $\mathbf{S}_{\mathbf{X}}$ is fixed. σ^{α} $(\alpha = x, y, z)$ are Pauli matrices, and N is the total number of lattice sites. The sign of the exchange coupling J depends on the system we consider; it is positive if the interaction is the *s*-*d* exchange as in case of canonical spin glasses, and is negative if it is the Hund-coupling as in case of manganites. The lifetime (τ) due to the scattering by normal impurities is included in the Green functions.

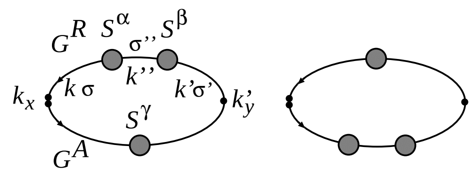

Fig. 1. Diagramatic representation of Hall conductivity. The interaction with the local spin is denoted by shaded small circles.

The anomalous Hall conductivity is calculated treating J as a perturbation. The first and second order contribution in H' vanish due to the spatial asymmetry of the current vertices. The chirality contribution comes at the third-order. The Hall conductivity is obtained after a straightforward calculation as

$$\sigma^{(3)}_{xy} = (4\pi)^2 \sigma_0 J^3 \nu^2 \tau \chi_0, \quad (2)$$

where σ_0 is the Boltzmann conductivity, $\sigma_0 \equiv \frac{N}{2V}\left(\frac{e}{m}\right)^2 \nu k_F^2 \tau$, and ν is the density of states per site (k_F is the Fermi wavenumber and V is the system volume). The uniform chirality χ_0 is given by

[1] E-mail:tatara@ess.sci.osaka-u.ac.jp

$$\chi_0 \equiv \frac{1}{N}\sum_{\mathbf{X}_i} \mathbf{S}_{\mathbf{X}_1}\cdot(\mathbf{S}_{\mathbf{X}_2}\times\mathbf{S}_{\mathbf{X}_3}) \times\left[\frac{(\mathbf{a}\times\mathbf{b})_z}{ab}I'(a)I'(b)I(c)+\frac{(\mathbf{b}\times\mathbf{c})_z}{bc}I(a)I'(b)I'(c) +\frac{(\mathbf{c}\times\mathbf{a})_z}{ca}I'(a)I(b)I'(c)\right], \quad (3)$$

where $\mathbf{X}_i$ runs over all the positions of local spins, while $\mathbf{a}\equiv\mathbf{X}_1-\mathbf{X}_2$, $\mathbf{b}\equiv\mathbf{X}_2-\mathbf{X}_3$ and $\mathbf{c}\equiv\mathbf{X}_3-\mathbf{X}_1$ are the vectors representing sides of the triangle ($a\equiv|\mathbf{a}|$ e.t.c.). $I(r)\equiv\frac{\sin k_F r}{k_F r}e^{-r/2\ell}$ and $I'(r)=\frac{dI(r)}{dr}$, where ℓ is elastic mean free path. It is seen that Hall current is driven by three spins which form a finite solid angle in spin space (i.e., finite local chirality $\chi_{123}\equiv\mathbf{S}_{\mathbf{X}_1}\cdot(\mathbf{S}_{\mathbf{X}_2}\times\mathbf{S}_{\mathbf{X}_3})$) spanning a finite area in coordinate space (as seen from $(\mathbf{a}\times\mathbf{b})_z$ etc.). As seen from the RKKY-type coupling, $I(r)$, contribution from largely separated three spins with the scale of r decays rapidly as $\sim e^{-3r/2\ell}/(k_F r)^3$, and the Hall effect is dominantly driven by chiralities of spins on small triangles. The expression of the uniform chirality, eq. (3), is a natural extension of the conventional (and naive) definition of the chirality in terms of spins on adjacent sites only.

At the lowest order, the chirality driven anomalous Hall effect is independent from the conventional spin-orbit contribution, and the total Hall resistivity ($\rho_{xy}=\sigma_{xy}\rho_0^2$) is simply their sum;

$$\rho_{xy}\simeq-\lambda M(A\rho_0+B\rho_0^2)+CJ^3\chi_0, \quad (4)$$

where λ is the spin-orbit coupling constant, M is the magnetization. A and B are independent of τ, while $C=\frac{1}{e^2}\frac{2V}{N}(\frac{m}{k_F})^2>0$ in our single band approximation. The sign of chirality contribution depends on whether the coupling is of the s-d type ($J>0$) or of the Hund type ($J<0$). It is seen that the three terms in eq. (4) depend differently on the impurity concentration. The chirality contribution is dominant in the clean regime and at low temperatures. It should be noted that the analysis in the strong Hund-coupling case which does not consider impurities yields $\rho_{xy}\propto\rho_0^2\chi$. These different dependences on ρ_0, which indicate different behavior as a function of temperature, would be useful in interpreting the experimental results. (One should note, however, that our perturbative expansion makes sense only if $J\tau\ll 1$.)

The chirality contribution to Hall coefficient is finite only if there is a net uniform chirality, $\chi_0\neq0$. Finite net chirality, however, may not be very easy to realize on regular lattices with simple nearest-neighbor exchange interaction, since the chirality on adjacent plaquettes usually tends to cancel each other due to symmetry. One possible mechanism to realize a finite net chirality has been proposed in Ref. [2], where it was argued in the strong coupling case that the spin-orbit interaction induced a net chirality in the presence of magnetization. The spin-orbit interaction can give rise to such "symmetry-breaking field" in the present weak coupling scheme, too. In fact, integrating out the electron in the presence of magnetization, the spin-orbit interaction at the second order in the exchange interaction (J) can be written as $\propto M\chi_0$[5]. This term is essentially the same as Dzaloshinsky-Moriya (DM) interaction, and is strongly dependent on the band structure.

Our theory is applicable to a wide class of magnetic systems including canonical spin glasses, in which the conduction electron is only weakly coupled to the local spin. In spin glasses, chirality order develops at low temperature leading to the spin-glass transition. Meanwhile, the chirality order there is spatially random without a uniform component. Even in this case, average of the squared chirality remains finite in small (mesoscopic) samples, and enhancement of the anomalous Hall coefficient is expected below the spin-glass transition. In fact, the sample-dependent or thermal-cycle-dependent fluctuations is given as $\sqrt{[(\delta\sigma_{xy})^2]_s}/\sigma_0\propto J^3\nu^2\tau\frac{1}{\sqrt{N}}n_{\rm m}^3(\ell/a_0)^4$, where $n_{\rm m}$ is the concentration of magnetic ions, a_0 being the lattice constant.

Without a direct method of magnetic detection of the chirality available so far, measurement of (fluctuation of) Hall conductivity would be powerful tool in experimental confirmation of the chirality ordering.

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