Reconsideration of the paramagnon theory in superfluid ^{3}He

Hiroaki Ikeda [1]

Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502

Abstract

The paramagnon theory in the superfluid ^{3}He is reconsidered. First, using the perturbation expansion up to the third order in the long range interaction related to the hard-core potential of ^{3}He, we evaluate the Gor'kov-Éliashberg equation. It is suggested that suppression of the density fluctuation in the high density leads to the p-wave triplet pairing. Second, we discuss the vertex correction for the paramagnon theory. Although the large mass enhancement in one paramagnon process at the nearly ferromagnetic quantum critical point drastically reduces the transition temperature, the vertex corrections suppress the mass enhancement, and stabilize the p-wave triplet pairing.

Key words: liquid ^{3}He; paramagnon theory; triplet superfluidity; long range interaction; vertex correction

Introduction— Recently, new materials indicated the superconducting transition have been successively found. Sr_2RuO_4 above all is in a triplet state in a quasi-two dimensional system, and has been investigated prosperously. Although the pairing mechanism has been considered as the paramagnon-like by analogy with ^{3}He at the beginning, the third order perturbation theory calculated by Nomura[1] is a leading candidate for the present. In this case, the main contribution to the triplet pairing is the third-order non-paramagnon process, including the Cooper loop. The perturbation theory naturally leads to the correct ground state at proper transition temperatures so far. This indicates that the superconducting transition in the strongly correlated electron systems originates in the repulsive interaction itself, not restricted within the spin fluctuation mechanism. Even in the on-site repulsion, the Fermi surface effect leads to the long range momentum dependence in the interaction between quasi-particles.[2] This general concept motivates us to improve the paramagnon thoery in the superfluid ^{3}He, which is typical in the strongly correlated Fermion systems. In fact, it is clear that the pairing mechanism in the superfluid ^{3}He is not simple paramagnon, since the liquid ^{3}He is not near the ferromagnetic quantum critical point (QCP), but rather in the almost localized Fermi liquid. We here reconsider the paramagnon theory in the superfluid ^{3}He.

Perturbation theory in finite range interaction— The bare interaction between ^{3}He atoms can be almost approximated by the hard-core potential with a radius of a. Since this potential is infinite within the radius a, we cannot simply use this potential in the perturbation expansion. In fact, however, this potential is more or less screened by other atoms in the Fermi sea, and the infinite-like behavior is cut by the finite value. Thus, we can approximate the interaction between ^{3}He by the finite-range potential $V(r) = U\theta(a-r)$, where $\theta(x)$ is the step funtion. In this case, the model Hamiltonian is described by

$$H = \sum_{p\sigma} \xi_p a^\dagger_{p\sigma} a_{p\sigma} + \sum_{pp'q\sigma\sigma'} V(q) a^\dagger_{p+q\sigma} a^\dagger_{p'-q\sigma'} a_{p'\sigma'} a_{p\sigma}, \quad (1)$$

where the notation is the conventional one, except that $\xi_p = p^2/2m - \mu$ represents the dispersion relation of ^{3}He. If r is discrete on lattice points with an unit larger than a, then this model Hamiltonian is reduced to the so-called Hubbard Hamiltonian. In the liquid ^{3}He, r is a continuous variable, and then the density occupied by ^{3}He is determined by the Fermi momentum p_F and

[1] E-mail: hiroaki@scphys.kyoto-u.ac.jp

the radius a. We hereafter set $p_F a = 2$ as the value of the liquid ^{3}He.

We now evaluate the Gor'kov-Éliashberg equation by the third-order perturbation expansion in this model Hamiltonian. Since the bare interaction possesses the momentum dependence, many complex diagrams, which vanish in the Hubbard model, appear in the pairing interaction and the normal self-energy. Fig. 2 shows the typical behavior of the anomalous self-energy $\Delta(p, \pi T)$ along the radial direction for the triplet p-wave pairing, where $\Delta(\boldsymbol{p}) = \Delta(p) Y_{1m}(\hat{p})$, and each parameter is $T = 0.01$, $V(q = 0) = 40$, $E_F = 0.35$ and $p_F = 0.58$. Here, positive values of $\Delta(p)$ indicate that the corresponding diagrams are attractive for the p-wave pairing. The second order $\Delta_2(p_F)$ and a part of the third order $\Delta_{3b}(p_F)$ are positive, while the remaining two parts of the third order $\Delta_{3a}(p_F)$ and $\Delta_{3c}(p_F)$ are relatively large negative. Finally, the total value $\Delta(p_F)$ up to the third order is negative, and then the triplet p-wave pairing does not be realized. In fact, the eigenvalue in the Gor'kov-Éliashberg equation is negative $\lambda = -0.01$. It should be noted that Δ_{3a}, which leads to the large negative value, is the interaction which contributes to the charge fluctuation. In the liquid ^{3}He, actually, the charge fluctuation is rather suppressed due to the hard-core potential and the high density. Thus, diagrams such as Δ_{3a} must be suppressed at low temperatures with all diagrams which contribute to the charge fluctuation. This implies the necessity of resummation of all order diagrams. Although Δ_{3c} which includes the Cooper loop has provided relatively large attractive interaction for the spin triplet pairing in Sr_2RuO_4, it here is negative. It may be affected by the resummation. Thus, it is implied that the pairing mechanism in Sr_2RuO_4 may be also modified by the resummation as considered here. On the other hand, Δ_2 attractive for the p-wave is included in the paramagnon process. Although Δ_{3b} is also attractive for the p-wave, this can be considered as the vertex correction for the one paramagnon process. Thus, this indicates that the vertex correction for the one paramagnon process is also large attractive for the p-wave pairing.

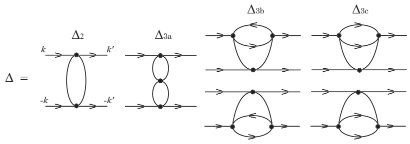

Fig. 1. Diagrams of the pairing interaction corresponding to each anomalous self-energy, Δ_2, Δ_{3a}, Δ_{3b}, and Δ_{3c}. A dot represents the symmetrized vertex of $V(q)$. Each diagram includes more complicated diagrams of $V(q)$.

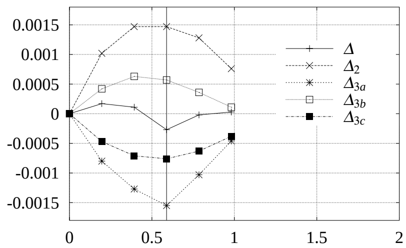

Fig. 2. Each anomalous self-energy $\Delta(p, \pi T)$ along the radial direction p. $\Delta(p_F)$, $\Delta_{3a}(p_F)$, $\Delta_{3c}(p_F) < 0$ and $\Delta_2(p_F)$, $\Delta_{3b}(p_F) > 0$ at $p_F = 0.58$.

Vertex corrections for paramagnon theory— We next evaluate the first-order vertex corrections for the paramagnon theory. For simplicity, $\chi(q)$ is calculated as $\chi_0(q)/(1 - U\chi_0(q))$ with the RPA. In the paramagnon theory, the pairing interaction is $U^2\chi(q)$. This interaction provides the eigenvalue $\lambda = 0.33$ for the p-wave pairing at $T = 0.01$, $E_F = 0.16$, $p_F = 0.39$, and $U = 90$, i.e., $1/(1 - U\chi_0(0)) = 10$. The first-order vertex corrections, that is, the second-order diagrams in $U^2\chi(q)$ are also attractive as expected above, and increase the eigenvalue into $\lambda = 0.43$. In addition, the mass enhancement factor is drastically suppressed from 3.5 to 2.0. This indicates that the vertex correction reduces the depairing effect, and stabilizes the p-wave triplet pairing. Thus, the paramagnon theory is over-simplified, and the effect of the vertex correction is rather important for stabilization of the p-wave pairing. We can expect that the p-wave triplet pairing is realized at relatively high temperatures, even far from the ferromagnetic QCP. This implies that the superfluidity of the actual liquid ^{3}He can be also lead to by the perturbation expansion in the effective interaction after resummation of all order diagrams such as indicated by Galitskii.[3]

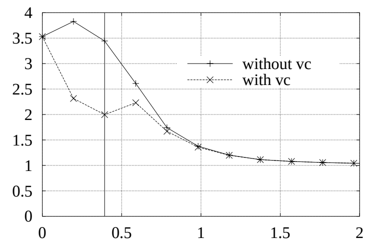

Fig. 3. Mass enhancement factor as a function of p. From 3.5 to 2.0 at $p_F = 0.39$ is reduced by the vertex correction.

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References

[1] T. Nomura *et.al.* J. Phys. Soc. Jpn. **69** (2000) 3678.

[2] W. Kohn *et.al.* Phys. Rev. Lett. **15** (1965) 524.

[3] V. M. Galitskii, Sov. Phys. JETP **34** (1958) 104.