Dynamical Conductivity and Localization Corrections in Small and Large Quantum Dots and Disordered Systems

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Abstract

Localization effect on the dynamical conductivity $\sigma(\omega)$ is examined in quantum dots and disordered electron systems. For the small volume limit, which is well described by the random matrix theory, $\mathrm{Re}[\sigma(\omega)]$ is known to be proportional to the DOS correlator $\langle\nu(E)\nu(E+\omega)\rangle$, hence to the universal two-level correlator $R_2(\omega/\Delta)$. Looking at localization effect, however, reveals the discrepancy between $\mathrm{Re}[\sigma(\omega)]$ and $\langle\nu(E)\nu(E+\omega)\rangle$, because only the former, not the latter, is consistent with the logarithmic weak localization correction on 2D and its RG treatment. We investigate and clarify the issue by evaluating the linear response $\sigma(\omega)$ directly from the nonlinear sigma formulation.

Key words: quantum dots; disordered systems; localization effect

1. Introduction

The dynamical conductivity $\sigma(\omega)$ for an isolated quantum dot characterizes the absorption of the material and can be obtained by either optical or transport measurements. For higher frequency, $\sigma(\omega)$ has a Drude form

$$\sigma(\omega) = \frac{\sigma_s(\omega)}{1 - i\omega\tau}. \tag{1}$$

$\sigma_s(\omega)$ is determined by the slow (diffusive) kinetics $\omega \lesssim 1/\tau$ and affected by the localization effect. In this paper, we clarify the interplay between quantum size effect observed typically in the region $\omega \lesssim \Delta$, and the weak localization effect upon $\sigma_s(\omega)$.

In an isolated small quantum dot with discrete energy levels, one needs to resort to some nonperturbative method to take account of the oscillating behavior for $\omega \lesssim \Delta$. In the small volume limit (the 0D limit), weak localization effect is negligible, so that the two-point DOS correlator is well described by the Wigner-Dyson's universal correlator $R_2^{\mathrm{WD}}(s = \frac{\omega}{\Delta})$,

$$\frac{1}{\Delta^2} R_2(s) := \langle\nu(E+\omega)\nu(E)\rangle \to \frac{1}{\Delta^2} R_2^{\mathrm{WD}}(s).$$

Besides, following the idea of the Gorkov-Eliashberg theory, the average of the matrix elements may be taken independently. If one assumes so, $\sigma_s(\omega)$ is expected to behave as

$$\mathrm{Re}\,\sigma_s(\omega) \approx \frac{\pi e^2 \langle v^2\rangle}{V} \langle\nu(E+\omega)\nu(E)\rangle = \sigma_0 R_2^{\mathrm{WD}}(s), \tag{2}$$

where σ_0 is the classical conductivity. Note that the above form consistently incorporates no absorption in an isolated system without any broadening for $\omega \ll \Delta$, with the classical value σ_0 expected for $\omega \gg \Delta$.

One may further conjecture that there exists some intermediate region that $\sigma_s(\omega)$ can be approximated by the DOS correlator $R_2(s)$ outside the strict 0D limit, if the system is so small that the localization correction gives little influence. If this were to be the case, the localization correction of $\mathrm{Re}\,\sigma_s(\omega)$ (especially for $\omega \lesssim \Delta$) should be more like that of $R_2(s)$ obtained in Ref. [1],

$$\frac{\mathrm{Re}\,\sigma_s(\omega)}{\sigma_0} \overset{(??)}{\approx} R_2(s) \approx 1 + \frac{1}{\beta}\Pi_2(\omega), \tag{3}$$

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$$\Pi_k(\omega) = \sum_q \Pi^k(q,\omega);\ \Pi(q,\omega) = \frac{1}{\pi g}\cdot\frac{L^{-2}}{q^2 - i\omega/D}, \quad (4)$$

Note that while $\Pi_1(\omega)$ diverges logarithmically for $d \to 2$, $\Pi_2(\omega)$ stay finite in this limit, so no logarithmic dependence expected on 2D.

The above surmised form of "the weak localization correction" in a small volume system strongly contrasts with the conventional weak localization perturbation results, which is applicable to $\omega \gg \Delta$, showing the logarithmic divergence on 2D.

$$\frac{\sigma_s(\omega)}{\sigma_0} = \begin{cases} 1 + \frac{2\epsilon}{d}\Pi_1^2(\omega) + \cdots & (\beta = 2) \\ 1 - \Pi_1(\omega) + \cdots & (\beta = 1) \end{cases}. \quad (5)$$

The issue we address here is to clarify whether such a non-standard weak localization effect suggested by Eq. (3) is possible in a non-perturbative region $\omega \lesssim \Delta$ or not. We directly evaluate $\sigma_s(\omega)$ by using the effective field theory (the supermatrix NL-σ model) to accommodate the nonperturbative effect correctly. It will be shown that the logarithmic behavior similar to Eq. (5) is *always* expected on 2D, irrespective of $\omega \gtrsim \Delta$ or $\omega \lesssim \Delta$, hence no behavior of Eq. (3). Though the assumption of the independent fluctuations between levels and matrix elements is appropriate at $g = \infty$, it fails even at very large g, which no longer leads to Eq. (3). Below we present all results for the unitary case ($\beta = 2$).

2. Results and conclusion

In order to treat the nonperturbative effect of ω, we use the supermatrix NL-σ model defined on G/K [2],

$$L[Q] = \sigma_0 L_0[Q] + i\omega\, L_\omega[Q], \quad (6)$$

$$L_0[Q] = \frac{1}{4}\mathrm{STr}\left[(\nabla Q)^2\right];\ L_\omega[Q] = \frac{\pi\nu}{2}\mathrm{STr}\left[Q\Lambda\right]. \quad (7)$$

We define and evaluate the observable conductivity $\sigma_s(\omega)$ by the response to the external (background) gauge field, following [3,4]. The effect of the background gauge field $A = g^{-1}\nabla g$ is incorporated into the action by the gauge transform of the Q-matrix, $Q \to gQg^{-1}$. Because of the symmetry of the theory, "the partition function" $Z[A]$ for any choice of external A should be of the form

$$Z[A] = \int DQ \exp\left(\int \sigma_0 L_0[gQg^{-1}] + i\omega L_\omega[gQg^{-1}]\right)$$
$$= \exp\left(\int \sigma_s(\omega) L_0[g\Lambda g^{-1}] + i\omega L_\omega[g\Lambda g^{-1}]\right),$$

whose RHS defines $\sigma_s(\omega)$ within the effective field theory, comparing between both sides up to $O(A^2)$. Next, following the prescription of Ref. [4], the choice of A is symmetrized to get a tractable formula. Note that unlike a replica NL-σ method, a careful attention is needed in symmetrization, which leads to a small yet important modification [5]. Eventually we can show that $\sigma_s(\omega)$ is expressed by

$$\frac{\sigma_s(\omega) - \sigma_0}{\sigma_0} = -\frac{1}{16V}\left\langle \int \mathrm{STr}^2(kQ)\right\rangle_Q$$
$$-\frac{\sigma_0}{32V}\left\langle \mathrm{STr}\left(\int kQ\partial_i Q \int kQ\partial_i Q\right)\right\rangle_Q \quad (8)$$

where $k = \mathrm{diag}(1,-1)$ in the BF block. The above expression of $\sigma_s(\omega)$ is valid within the same validity of the supermatrix theory, which allows us to apply any nonperturbative evaluation of the Q-matrix integral. Whereas the first local contribution of Eq. (8) was missing in the replica method, its presence is of critical importance for our purpose here. It shows a direct connection between $\sigma_s(\omega)$ and the inverse participation ratio, as well as being responsible for reproducing $R_2^{\mathrm{WD}}(s)$ in the 0D limit [5].

A reliable estimate of the weak localization correction for a small volume system can be obtained by taking account of the weak localization effect around the 0D limit [6]. After some calculations, the result for the unitary case is summarized as follows.

$$\frac{\sigma_s(\omega)}{\sigma_0} = \mathcal{R}(s)\left[1 + \frac{2\epsilon}{d}\Pi_1^2(\omega) - \frac{4(d+3)}{d}\Pi_2(\omega)\right]$$
$$-\frac{2}{i\pi s}\frac{\epsilon}{d}\Pi_1(\omega) + \left[\frac{4\epsilon}{d}s\mathcal{R}'(s) + s^2\mathcal{R}''(s)\right]\Pi_2(\omega) + \cdots (9)$$

where we introduce the complexified $R_2(s)$ function as

$$\mathcal{R}(s) := 1 + \frac{e^{2i\pi s} - 1}{2\pi^2 s^2};\quad \mathrm{Re}\left[\mathcal{R}(s)\right] = R_2^{\mathrm{WD}}(s). \quad (10)$$

It shows that the weak localization effect is always dominated by $\epsilon\Pi_1^2(\omega)$. Since the ϵ-factor is canceled out partially with the dimensional pole ϵ^{-2} from Π_1^2, it gives a usual logarithmic behavior of the order of $O(1/g^2)$ in the unitary case, and all the nonperturbative effect of ω is incorporated in $\mathcal{R}(s)$.

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