

# Monte Carlo Study of Pseudo-Gap Temperature $T^*$ within JJA Model

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## Abstract

We study pseudo-gap temperature  $T^*$  of high- $T_c$  superconductors by a Monte Carlo simulation of anisotropic 3D Josephson Junction Array (JJA) model based on the Ginzburg-Landau theory. We investigate  $T^*$  both in the cases of zero external current and finite external current  $I$  in the JJA. It is found that, the external current  $I$  depresses only a little the pseudo-gap temperature  $T^*$ , while the superconducting critical temperature  $T_c$  is much affected by  $I$ .

*Key words:* Pseudo-gap temperature, High- $T_c$  cuprate superconductor, Josephson Junction Array model, Ginzburg-Landau theory

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Much attention has been focused on the pseudo gap of high- $T_c$  cuprate superconductors. On the basis of the Josephson Junction Array (JJA) model for the high- $T_c$  cuprate superconductors [1-4], we have investigated the pseudo-gap temperature  $T^*$  and the superconducting critical temperature  $T_c$ . In this paper, we report our result of the Monte Carlo simulation for the effect of the external current  $I$  on  $T^*$  and  $T_c$ .

We model the ceramic high- $T_c$  materials as a JJA which consists of weakly coupled superconducting grains on an anisotropic 3D lattice (i.e., a stack of 2D-lattice layers) [1-4]. The grain at the lattice site  $i$  is characterized by the phase  $\theta_i$  and the amplitude  $|\phi_i|$  of the superconducting order parameter  $\phi_i = |\phi_i| \exp(i\theta_i)$ .

In previous papers [5,6], we performed the Monte Carlo simulation with a Hamiltonian in which only the phase  $\theta_i$  was taken into account, and could investigate the effect of  $I$  on  $T_c$  only. In this paper, we investigate not only  $T_c$  but also  $T^*$  by considering both the phase

$\theta_i$  and the amplitude  $|\phi_i|$ . Our effective Hamiltonian is given as

$$F_{\text{eff}} = F(\{\theta_i, |\phi_i|\}) - T \sum_i \ln(|\phi_i|) \\ = F_0 + F_1 - T \sum_i \ln(|\phi_i|), \quad (1)$$

where

$$F_0 = - \sum_{i,j} |\phi_i|^2 [\cos(\theta_i - \theta_j) - 1] \\ - \alpha \sum_{i,k}^{\prime} |\phi_i|^2 [\cos(\theta_i - \theta_k) - 1] \\ - \sum_{i,j} I \cdot [\theta_i - \theta_j], \quad (2)$$

$$F_1 = \frac{1}{2} \sum_{i,j} (|\phi_i| - |\phi_j|)^2 \\ + \frac{\alpha}{2} \sum_{i,k}^{\prime} (|\phi_i| - |\phi_k|)^2$$

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$$+ \frac{A}{2} \sum_i [ - (1 - T/T_0) |\phi_i|^2 + \frac{1}{2} |\phi_i|^4 ]. \quad (3)$$

Here,  $\sum_{i,j}$  means the summation over the neighboring intra sites in a 2D layer and  $\sum'_{i,k}$  over the neighboring inter sites between the layers. The parameter  $\alpha$  of the system anisotropy corresponds to  $\Gamma^2$  of Ref. [7];  $\alpha \rightarrow 1$  (3D limit) and  $\alpha \rightarrow 0$  (2D limit). The parameter  $A$  corresponds to  $(a_{\parallel}/\xi_{\parallel})^2$  and  $T_0$  corresponds to  $T'_{\text{MF}}$  [7].  $T$  is the temperature. The external current  $I$  is defined in Refs. [5,6].

We perform the Monte Carlo simulation on the 3D JJA system [Eqs. (1)–(3)] with  $A = 1$  and the anisotropy ratio  $\alpha = 0.01$ . The system size is  $20 \times 20 \times 20$  with periodic boundary conditions.  $T_c$  is defined as the temperature at which the susceptibility  $\chi = \sum_{i,j} \langle \cos \theta_i \cos \theta_j \rangle$  diverges [5,6]. The symbol  $\langle \dots \rangle$  represents the statistical average.  $T^*$  is defined as the temperature at which  $\sum_i \langle |\phi_i| \rangle = 0$ . In Table 1, we show the result obtained by the Monte Carlo Simulation. It is noticeable that, while the superconducting critical temperature  $T_c$  is much affected by  $I$  (namely,  $\sim 50\%$  decrease of  $T_c$ ), the external current  $I$  depresses only a little the pseudo-gap temperature  $T^*$ . We have also performed the simulation with  $\alpha = 0.001$  and  $10^{-4}$ , and found that the results depended only a little on the anisotropy ratio  $\alpha$ .

We hope that this result (i.e., the difference in the  $I$  sensitivity between  $T^*$  and  $T_c$ ) can be observed experimentally by applying the external current to the high- $T_c$  cuprate superconductors and simultaneously measuring the pseudo-gap temperature and the superconducting critical temperature. Such observations are expected to be helpful to identify the origin of the pseudo gap in the high- $T_c$  cuprates, i.e., to identify whether or not the separation between the pseudo-gap temperature and the superconducting critical one is described by the anisotropic 3D JJA- and XY(phase)-model scenarios [1–8] for the superconductivity in the high- $T_c$  cuprates.

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Table 1

The pseudo gap temperature  $T^*$  and the superconducting critical temperature  $T_c$  vs the external current  $I$  obtained by the Monte Carlo Simulation. The parameter  $A = 1$ . The anisotropy ratio  $\alpha = 0.01$ .

$I$	0	0.5	1.0
$T^*$	0.26	0.24	0.22
$T_c$	0.09	0.09	0.05

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