Neutral Collective Excitations in Striped Hall States

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Abstract

In the striped Hall state, a magnetic translation in one direction is spontaneously broken to the discrete translation. The spectrum of the neutral collective excitation is obtained in the single mode approximation at half-filled third and fourth Landau levels. The spectrum is anisotropic and has a multiple line node structure. In one direction, the spectrum resembles the liquid Helium spectrum with the phonon and roton minimum.

Key words: quantum Hall effect; striped state; collective excitation; single mode approximation

Recently, highly anisotropic states were observed around the half-filled third and higher Landau levels[1,2]. The anisotropic state is believed to be the striped Hall state which is a unidirectional charge density wave in a mean field theory[3,4]. The anisotropy is naturally explained by the anisotropic Fermi surface in the magnetic Brillouin zone[5]. It is predicted that the fluctuation effect turns the striped state into the smectic or nematic liquid crystal[6].

In this paper, we investigate the property of the neutral collective excitations in the striped Hall state. In the absence of edges and disorder, a two-dimensional electron system under a uniform magnetic field has the magnetic translation and rotation symmetry. In the striped Hall state, a magnetic translation in one direction is spontaneously broken to the discrete translation and the rotation is also spontaneously broken to the π-rotation. Goldstone theorem for the striped Hall state[7] says that the gapless excitation exists in the neutral charge sector and couples with the density operator. The spectrum of the neutral collective excitation is obtained in the single mode approximation numerically. We use the unit $\hbar = c = 1$ and $a = \sqrt{2\pi\hbar/eB} = 1$.

In a strong magnetic field B, the free kinetic energy is quenched. Therefore we study only the interaction Hamiltonian projected into the l th Landau level, that is

$$H_l = \frac{1}{2}\int d\mathbf{r}d\mathbf{r}'\Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\Psi(\mathbf{r}')\Psi(\mathbf{r}),$$
$$\Psi(\mathbf{r}) = \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} b_l(\mathbf{p})\langle \mathbf{r}|l,\mathbf{p}\rangle, \qquad (1)$$

where Ψ is the electron field operator projected into the l th Landau level[5], BZ stands for Brillouin zone $|p_i| < \pi$, and $V(\mathbf{r}) = q^2/r$ ($q^2 = e^2/4\pi\epsilon$, ϵ is the dielectric constant). $b_l(\mathbf{p})$ is an annihilation operotor for one-particle state $\langle \mathbf{r}|l,\mathbf{p}\rangle$ which is a Bloch wave on the magnetic von Neumann lattice[5].

The mean field state for the striped Hall state is constructed as

$$|\mathrm{stripe}\rangle = \prod_{\mathbf{p}\in \mathrm{F.S.}} b_l^\dagger(\mathbf{p})|0\rangle, \qquad (2)$$

where F.S. means Fermi sea $|p_y| < \pi/2$, and $|0\rangle$ is the state in which the $l-1$ th and lower Landau levels are fully occupied. The charge density for state of Eq. (2) is uniform in y direction and periodic in x direction. The period r_s is a parameter of the von Neumann lattice and is fixed by the minimum energy condition[5].

We calculate the spectrum for a neutral collective excitation at the half-filled third and fourth Landau level using the single mode approximation. The single mode

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approximation is successful in the FQHS because the backflow problem is absent for the electron states projected to the Landau level[8]. Projected density operator $\rho(\mathbf{k})$ is written as $e^{-k^2/8\pi}L_l(k^2/4\pi)\rho_*(\mathbf{k})$, where

$$\rho_*(\mathbf{k}) = \int_{\mathrm{BZ}} \frac{d^2p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) b_l(\mathbf{p}-\hat{\mathbf{k}}) e^{-\frac{i}{4\pi}\hat{k}_x(2p_y-\hat{k}_y)}, \quad (3)$$

where $\hat{\mathbf{k}} = (r_s k_x, k_y/r_s)$. It is well-known that the density operators projected to the Landau level are non-commutative,

$$[\rho_*(\mathbf{k}), \rho_*(\mathbf{k}')] = -2i \sin\left(\frac{\mathbf{k}\times\mathbf{k}'}{4\pi}\right)\rho_*(\mathbf{k}+\mathbf{k}'). \quad (4)$$

The variational excited state is defined by $|\mathbf{k}\rangle = \rho_*(\mathbf{k})|\mathrm{stripe}\rangle$ and the variational excitation energy $\Delta(\mathbf{k})$ is written as

$$\begin{aligned}\Delta(\mathbf{k}) &= \frac{\langle\mathbf{k}|(H_l - E_0)|\mathbf{k}\rangle}{\langle\mathbf{k}|\mathbf{k}\rangle} = \frac{f(\mathbf{k})}{s(\mathbf{k})},\\ f(\mathbf{k}) &= \langle 0|[\rho_*(-\mathbf{k}), [H_l, \rho_*(\mathbf{k})]]|0\rangle/2N_e^*, \quad (5)\\ s(\mathbf{k}) &= \langle 0|\rho_*(-\mathbf{k})\rho_*(\mathbf{k})|0\rangle/N_e^*,\end{aligned}$$

where E_0 is a ground state energy, N_e^* is a electron number in the l th Landau level, and $s(\mathbf{k})$ is the so-called static structure factor. To derive these expressions, we use the relation $f(-\mathbf{k}) = f(\mathbf{k})$ and $s(-\mathbf{k}) = s(\mathbf{k})$ due to π rotation symmetry. Using the commutation relation (4), $f(\mathbf{k})$ is written as

$$\begin{aligned}f(\mathbf{k}) = 2\int \frac{d^2k'}{(2\pi)^2} v_l(k') \sin^2\left(\frac{\mathbf{k}'\times\mathbf{k}}{4\pi}\right) \quad (6)\\ \times\{s(\mathbf{k}+\mathbf{k}') - s(\mathbf{k}')\},\end{aligned}$$

where $v_l(k) = e^{-k^2/4\pi}(L_l(k^2/4\pi))^2 2\pi q^2/k$. Therefore the variational excitation energy is calculable if we know the static structure factor $s(\mathbf{k})$. For the mean field state (2), $s(\mathbf{k})$ behaves as $|k_y|/\pi r_s$ at small k_y and periodic in k_y direction with a period $2\pi r_s$. The numerical results for the energy spectrum Δ for $\nu = l + 1/2$, $l = 2$ and 3 are shown in Figs. 1 and 2, respectively.

As seen in these figures, the spectrum in k_y direction resembles the liquid Helium spectrum with the phonon and roton minimum. The spectrum has a multiple line node at $k_y = 2\pi r_s n$ (n is integer). The comparison to particle-hole excitation energy[7] shows that the single mode approximation is good around $k_y = 2\pi r_s n$. We hope these spectrum will be observed for the evidence of the striped Hall state.

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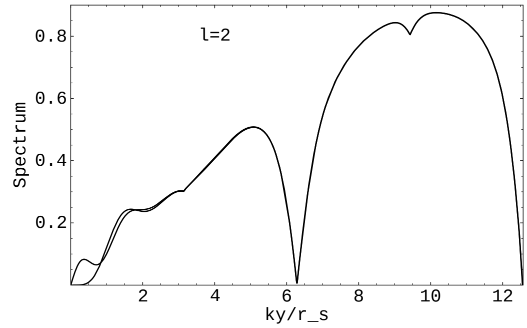

Fig. 1. Energy spectrum Δ at $0 < r_s k_y < 4\pi$ for $k_x = 0$ and 1 (linear dispersion at $k_y = 0$), $\nu = 2 + 1/2$ in the single mode approximation. The unit of $\mathbf{k}$ is a^{-1} and the unit of spectrum is q^2/a. The same unit is used in Fig. 2.

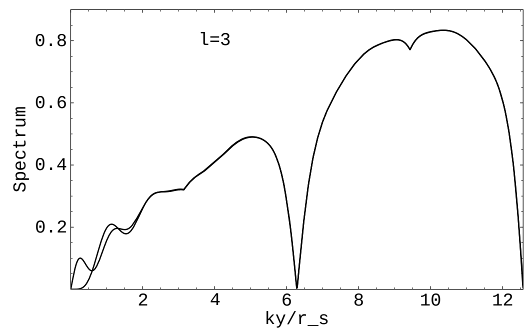

Fig. 2. Energy spectrum Δ at $0 < r_s k_y < 4\pi$ for $k_x = 0$ and 1 (linear dispersion at $k_y = 0$), $\nu = 3 + 1/2$ in the single mode approximation. The unit of $\mathbf{k}$ is a^{-1} and the unit of spectrum is q^2/a.

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