

# New varieties of order parameter symmetry in quasi-one-dimensional superconductors

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## Abstract

The symmetries of the superconducting order parameter in quasi-one-dimensional systems are investigated by the renormalization group method in a model characterized by backward scattering  $g_1$  and forward scattering  $g_2$ . Various types of pairing can be realized for different sets of  $(g_1, g_2)$ . In the case of  $g_1 = g_2$  (Hubbard model), for example, the transition temperature of  $p$ -like triplet becomes higher than that of  $d$ -like singlet.

*Key words:* unconventional superconductivity, quasi-one dimension,  $(\text{TMTSF})_2\text{PF}_6$

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Recent experiments on quasi-one-dimensional (q1d) superconductor  $(\text{TMTSF})_2\text{PF}_6$  are deeply fascinating. The upper critical field  $H_{c2}$  along the  $b$ -axis exceeding the Clogston limit [1] and the unchanged Knight shift through  $T_c$  [2] strongly suggest the triplet pairing. Then, its superconducting mechanism is wrapped in mystery, since spin singlet pairings are generally believed to realize in the vicinity of SDW phase from the studies on heavy electron superconductivity [3,4] or high- $T_c$  cuprates[5].

In this paper, we investigate the pairing mechanism and the order parameter symmetry in q1d systems. We first construct a pairing interaction applying the renormalization group (RG) method and then solve the gap equation to determine the superconducting gap. This treatment makes it possible to take into account the characteristics of q1d systems which cannot be captured by the so-called FLEX approximation.[6]

The interaction part of the Hamiltonian can be written in the form

$$\mathcal{H}_{\text{int}} = \frac{1}{L} \sum_{k_i, \alpha, \beta} g_1 a_{k_1 \alpha}^\dagger b_{k_2 \beta}^\dagger a_{k_3 \beta} b_{k_4 \alpha}$$

$$+ \frac{1}{L} \sum_{k_i, \alpha, \beta} g_2 a_{k_1 \alpha}^\dagger b_{k_2 \beta}^\dagger b_{k_3 \beta} b_{k_4 \alpha},$$

where  $a_{k, \alpha}^\dagger, a_{k, \alpha}$  ( $b_{k, \alpha}^\dagger, b_{k, \alpha}$ ) denotes the operators for the electrons belonging to the branch containing the Fermi point  $+k_F$  ( $-k_F$ ). Here,  $g_1$  denotes the interaction with the momentum transfer near  $2k_F$ ,  $g_2$  with the momentum transfer near 0. Following the usual RG method[7], let us write the four-point vertex  $\Gamma_i^*$  irreducible with respect to the particle-particle channel in the form:  $\Gamma_{\alpha \beta \gamma \delta}^*(q) = g_1 \tilde{\Gamma}_1^*(q) \delta_{\alpha \gamma} \delta_{\beta \delta} - g_2 \tilde{\Gamma}_2^*(q) \delta_{\alpha \delta} \delta_{\beta \gamma}$ , where  $|q - 2k_F| = |k_1 - k_3|$ . Up to the second-order (namely, the first-order correction), they are calculated as

$$\tilde{\Gamma}_1^*(q) = 1 + \frac{1}{\pi v_F} (g_1 - g_2) [\ln(q/k_0) - i\pi/2] \quad (1)$$

$$\tilde{\Gamma}_2^*(q) = 1 - \frac{1}{2\pi v_F} g_2 [\ln(q/k_0) - i\pi/2] \quad (2)$$

They obey the scaling relation  $\tilde{\Gamma}_i^*[(q/k'_0), g'_1, g'_2] = z_i(k'_0/k_0) \tilde{\Gamma}_i^*[(q/k_0), g_1, g_2]$ . Using the results of first-order renormalization [7], the scaling equations can be solved as

$$\tilde{\Gamma}_1^*(q) = [1 - (g_1/\pi v_F) \ln(q/k_0)]^{-1/2} (q/k_0)^\alpha, \quad (3)$$

$$\tilde{\Gamma}_2^*(q) = [1 - (g_1/\pi v_F) \ln(q/k_0)]^{1/4} (q/k_0)^{\alpha/2}, \quad (4)$$

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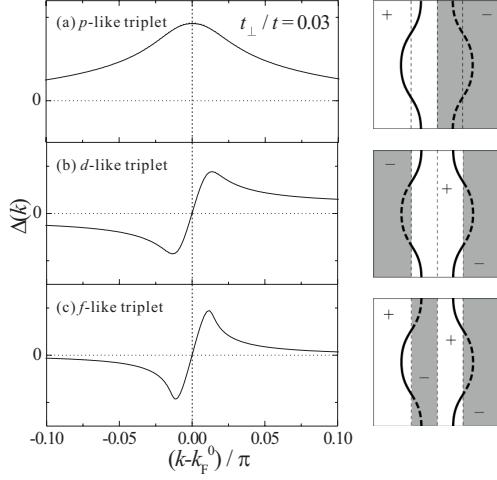


Fig. 1. Superconductive gap  $\Delta(k)$  as a function of  $k$  (left panels).  $k_F^0 = \pi/4$  is the Fermi surface in the absence of  $t_{\perp}$ . The right panels show the Fermi surface, where the solid (dashed) line indicates  $\Delta(k) > 0$  ( $< 0$ ).

where  $\alpha = (g_1/2) - g_2 < 0$ . The total irreducible part can be separated into even (+) and odd (−) part as  $\Gamma^*(q) = g_1\Gamma_1^*(q) \pm g_2\Gamma_2^*(q)$ . It is important to note that the response function for the charge and the spin density have the same exponent  $\alpha$ . In the presence of  $t_{\perp}$ , in q1d sysym, the transition temperature of CDW and SDW are easily suppressed by the interchain hopping, while  $T_c$  of the present superconductivity is not[8]. Therefore, with a moderate interchain hopping  $t_{\perp}$ , the superconductivity mediated by  $\Gamma^*(q)$  is realized instead of CDW or SDW for  $\alpha < 0$ .

The transition temperature  $T_c$  and the order parameter  $\Delta(k)$  of the superconductivity are determined by the following linearized gap equation

$$\Delta(k) = - \sum_{k', k'_{\perp}} \Gamma^* (|k - k'| - 2k_F) \frac{\Delta(k')}{2\xi_{k', k'_{\perp}}} \times \tanh \frac{\xi_{k', k'_{\perp}}}{2k_B T_c}. \quad (5)$$

For  $g_1 - 2g_2 < 0$ , the obtained order parameters are roughly classified into three types. 1) triplet order parameter with finite gap at  $k_F^0 = \pi/4$  (*p*-like triplet); 2) singlet order parameter with node just at  $k_F^0$  (*d*-like singlet); 3) triplet order parameter with node just at  $k_F^0$  (*f*-like triplet). Each type is shown in Fig. 1. With the present pairing interaction, the gap function depends only on  $k$ , though the dispersion depends both on  $k$  and  $k_{\perp}$ , and the Fermi surface is slightly warped due to the interchain hopping. Therefore, *d*- and *f*-like gap function vanishes on line  $k = \pi/4$ , and they have four

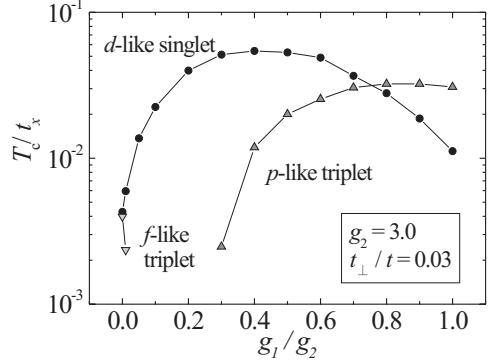


Fig. 2. Transition temperature  $T_c$  for different sets of  $g_1/g_2$  ( $g_2 = 3.0$ ).

line nodes on the Fermi surface (see the right panels of Fig. 1).

The calculated  $T_c$ 's are shown in Fig. 2. For  $g_1=0$ , the transition temperature of *d*-like singlet and that of *f*-like triplet are almost degenerate. In the case of  $g_1 = g_2$ , corresponding to the Hubbard model,  $T_c$  of *p*-like triplet is higher than that of *d*-like singlet. In the intermediate region,  $0.5 < g_1/g_2 < 1.0$ ,  $T_c$  of *d*-like singlet and that of *p*-like triplet are comparable.

It is natural to expect that real q1d conductors lie in the region  $g_1 < g_2$  rather than  $g_1 = g_2$  (Hubbard model) due to long-range Coulomb interaction. Assuming that the coupling constants are in the region  $0.5 < g_1/g_2 < 1.0$  for  $(\text{TMTSF})_2\text{PF}_6$ , we may conclude that the *d*-like singlet and *p*-like triplet are competing in this material.

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